Nonlinear Analysis 74 (2011) 5544–5553

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Positive solutions of Lane–Emden systems with negative exponents: Existence, boundary behavior and uniqueness☆

Zhijun Zhang*

School of Mathematics and Information Science, Yantai University, Yantai 264005, Shandong, PR China

ARTICLE INFO

Article history:
Received 19 October 2010
Accepted 13 May 2011
Communicated by Enzo Mitidieri

Keywords:
Semilinear elliptic systems
Singular terms
Dirichlet problems
Existence
Boundary behavior
Uniqueness

ABSTRACT

We study the existence, boundary behavior and uniqueness of solutions for the singular elliptic system $-\Delta u = u^{-p}v^{-q}$, $-\Delta v = u^{-r}v^{-s}$, $u > 0$, $v > 0$, $x \in \Omega$, $u|_{\partial\Omega} = v|_{\partial\Omega} = 0$, where Ω is a bounded domain with smooth boundary in $\mathbb{R}^N$, $p, s \geq 0$ and $q, r > 0$. Our results are obtained in a range of p, q, r, s different from those in [M. Ghergu, Lane–Emden systems with negative exponents, J. Funct. Anal. 258 (2010) 3295–3318].

1. Introduction

Consider the following singular semilinear elliptic system:

$$\begin{cases} -\Delta u = u^{-p}v^{-q}, & \text{in } \Omega, \\ -\Delta v = u^{-r}v^{-s}, & \text{in } \Omega, \\ u > 0, \quad v > 0, & \text{in } \Omega, \\ u|_{\partial\Omega} = v|_{\partial\Omega} = 0, \end{cases} \tag{1.1}$$

where Ω is a bounded domain with smooth boundary in $\mathbb{R}^N$, $p, s \geq 0, q, r > 0$.

In [1], Ghergu first established the existence, non-existence, C^1-regularity and uniqueness of classical solutions (in $C^2(\Omega) \cap C(\bar{\Omega})$) in terms of p, q, r and s. Specifically, he showed the following results.

Lemma 1.1 (*Non-Existence*). *Let p, q, r, s satisfy one of the following conditions:*

(i) $r \min\{1, \frac{2-q}{1+p}\} \geq 2$;
(ii) $q \min\{1, \frac{2-r}{1+s}\} \geq 2$;
(iii) $q > \max\{1,\ r-1\}, 2r > (1-s)(1+p)$ *and* $q(1+p-r) > (1+p)(1+s)$;
(iv) $s > \max\{1,\ q-1\}, 2q > (1-p)(1+s)$ *and* $r(1+s-q) > (1+p)(1+s)$.

Then the system (1.1) has no solutions.

☆ This work is supported in part by NNSF of PR China under grant 10671169 and by Shandong Province Natural Science Foundation under grant number 2009ZRB01795.

* Tel.: +86 535 6902406.
E-mail address: zhangzj@ytu.edu.cn.

0362-546X/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2011.05.038