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Positive solutions of asymptotically linear Schrödinger–Poisson systems with a radial potential vanishing at infinity

Juntao Sun [a,*], Haibo Chen [a], Liu Yang [a,b]

[a] Department of Mathematics, Central South University, Changsha, 410075 Hunan, People's Republic of China

[b] Department of Mathematics, Hengyang Normal University, Hengyang, 421008 Hunan, People's Republic of China

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ABSTRACT

In this paper, we study the Schrödinger–Poisson system

$$\begin{cases} -\Delta u + V(x)u + \lambda\phi(x)u = K(x)f(u), & \text{in } \mathbb{R}^3, \\ -\Delta\phi = u^2, \quad u > 0, & \text{in } \mathbb{R}^3, \end{cases} \tag{SP}$$

and prove the existence of positive solutions for system (*SP*) when the nonlinearity f has growth at most linear for λ small, allowing the potential $V(x)$ to vanish at infinity. In addition, also we obtain the nonexistence of a nontrivial positive solution for $\lambda \geq \frac{1}{4}$.

1. Introduction

Consider the following nonlinear system:

$$\begin{cases} -\Delta u + V(x)u + \lambda\phi(x)u = K(x)f(u), & \text{in } \mathbb{R}^3, \\ -\Delta\phi = u^2, \quad u > 0, & \text{in } \mathbb{R}^3, \end{cases} \tag{SP}$$

where $\lambda > 0$ is a parameter, and $V, K : \mathbb{R}^N \to \mathbb{R}^+$ are radial and smooth. Such a system, also known as the nonlinear Schrödinger–Maxwell system, arises in an interesting physical context. Indeed, according to a classical model, the interaction of a charged particle with an electromagnetic field can be described by coupling the nonlinear Schrödinger and the Poisson equations (we refer the reader to [1] for more details on the physical aspects). In particular, if we are looking for electrostatic-type solutions, we just have to solve (*SP*). To be precise, we will find solutions for (*SP*) with the following properties:

$$u \in H^1(\mathbb{R}^3), \quad u > 0, \qquad \lim_{|x|\to\infty} u(x) = 0. \tag{1.1}$$

Variational methods and critical point theory are powerful tools in studying nonlinear differential equations [2–4], and in particular Hamiltonian systems and elliptic equations [5–10]. In recent years, system (*SP*) has been studied widely via modern variational methods under the various hypotheses; see [11–18] and the references therein. More precisely, Ruiz [14] obtained the existence and nonexistence of radial solutions for (*SP*) with $V = K = 1$. Soon after, Ambrosetti and Ruiz [11] obtained multiplicity results for (*SP*) with $V = K = 1$.

* Corresponding author.
E-mail addresses: sunjuntao2008@163.com (J. Sun), math_chb@mail.csu.cn (H. Chen), yangliu19731974@yahoo.com.cn (L. Yang).

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