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Robinson metric regularity of parametric variational systems☆

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ABSTRACT

In this paper, the Robinson metric regularity of a parametric variational system is investigated. Some applications to the contingent derivative of parametric variational system and to the Robinson metric regularity of a parametric vector optimization problem are then studied.

1. Introduction

In this paper we study the Robinson metric regularity (see Definition 2.4) of solutions to variational systems defined by parametric generalized equations

$$f(x, y) \in Q(x, y)$$

with the decision variable $y \in Y$ and the parameter $x \in X$, where $f : X \times Y \to Z$ is a vector-valued map, $Q : X \times Y \rightrightarrows Z$ is a set-valued map, X and Y are linear metric spaces whose metrics will both be denoted by d and Z is a normed space with the norm $\|\cdot\|$. Notice that the above parametric generalized equation includes any parametric generalized equation of the following form

$$f_1(x, y) \in Q_1(x, y), \quad \text{with } y \in K(x),$$

since we could define $f(x, y) := (f_1(x, y), y)$ and $Q(x, y) := Q_1(x, y) \times K(x)$.

Generalized equations were introduced by Robinson [1] and have been widely recognized as a convenient model for the unified study in many optimization-related areas including variational inequalities, optimal control, mathematical economics, equilibrium, vector optimization problems, etc. In particular, parametric generalized equations reduce to parametric vector optimization problems (for short, PVOPs) (see Section 4.2).

The solution map associated to a parametric generalized equation is the set-valued map $S : X \rightrightarrows Y$ defined by

$$\text{(PVS)} \quad S(x) := \{y \in Y : f(x, y) \in Q(x, y)\},$$

which is called a parametric variational system (for short, PVS), where S is also called an implicit multifunction. Recently, Aragón Artacho and Mordukhovich [2] investigated the metric regularity and the Lipschitz-likeness of a special PVS where Q only depends on the decision variable y. In [3], Uderzo discussed the global and local criteria for Lipschitzian property and metric regularity of PVS. Moreover, from the standpoint of coderivative analysis Levy and Mordukhovich [4] and

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