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Existence of a nontrivial solution for a class of hemivariational inequality problems at double resonance☆

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ABSTRACT

In this paper, we discuss a class of semilinear elliptic hemivariational inequality problems. By using the nonsmooth minimax principle for locally Lipschitz functions, we establish the existence of a nontrivial solution for the semilinear elliptic hemivariational inequality problem, where incomplete double resonance occurs at infinity between two distinct consecutive eigenvalues.

1. Introduction

In this paper, we are concerned with the following semilinear elliptic problem with a nonsmooth potential:

$$\begin{cases} -\Delta u \in \partial F(x, u), & \text{a.e } x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subseteq \mathbb{R}^N (N \geq 3)$ is a bounded domain with smooth boundary $\partial\Omega$, and the potential function $F(x, u) : \Omega \times \mathbb{R} \to \mathbb{R}$ is measurable with respect to x and locally Lipschitz with respect to u. $\partial F(x, u)$ denotes the Clarke generalized gradient of the locally Lipschitz function $u \mapsto F(x, u)$ (see Section 2 for more details). Problems like (1.1) are known as elliptic hemivariational inequalities (or variational inclusion problems).

Hemivariational inequality problem is a new type of inequality problem which has been introduced by Panagiotopoulos (cf. [1]) in order to deal with problems arise from mechanics and engineering whose variational forms are such inequalities which express the principle of virtual work or power. Since that, the hemivariational inequalities have been investigated by a number of authors, the reader is referred to [2–10] and the references therein, where the treatment relies on topological method and nonsmooth critical point theory.

When the potential function $F(x, u)$ is a primitive of a continuous function $f(x, u) : \Omega \times \mathbb{R} \to \mathbb{R}$, then $\partial F(x, u) = \{f(x, u)\}$ and Problem (1.1) will become the following "smooth" semilinear Dirichlet boundary value problem:

$$\begin{cases} -\Delta u = f(x, u), & x \in \Omega, \\ u = 0, & x \in \partial\Omega. \end{cases} \tag{1.2}$$

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