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Lipschitz-like property of an implicit multifunction and its applications ☆

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ABSTRACT

The aim of this work is twofold. First, we use the advanced tools of modern variational analysis and generalized differentiation to study the Lipschitz-like property of an implicit multifunction. More explicitly, new sufficient conditions in terms of the Fréchet coderivative and the normal/Mordukhovich coderivative of parametric multifunctions for this implicit multifunction to have the Lipschitz-like property at a given point are established. Then we derive sufficient conditions ensuring the Lipschitz-like property of an efficient solution map in parametric vector optimization problems by employing the above implicit multifunction results.

1. Introduction

The paper mainly deals with the stability theory of implicit multifunctions and parametric vector optimization problems. We first give some notation and definitions.

Let X, Y be Banach spaces and (P, d) be a metric space, and let $F : P \times X \rightrightarrows Y$ be a parametric multifunction. By means of this parametric multifunction one can define an *implicit multifunction* $G : P \rightrightarrows X$ as follows:

$$G(p) := \{x \in X \mid 0 \in F(p, x)\}. \tag{1.1}$$

Let $K \subset Y$ be a pointed, closed and convex cone with an apex at the origin.

Definition 1.1. We say that $y \in A$ is an *efficient point* of a subset $A \subset Y$ with respect to K if and only if $(y - K) \cap A = \{y\}$. The set of efficient points of A is denoted by $\text{Eff}_K A$. We stipulate that $\text{Eff}_K \emptyset = \emptyset$.

Given a vector function $f : P \times X \to Y$, we consider the following *parametric vector optimization problem*:

$$\text{Eff}_K \left\{ f(p, x) \mid x \in X \right\}, \tag{1.2}$$

where x is the *unknown* (decision variable) and $p \in P$ a *parameter*.

For each $p \in P$, we put

$$\mathcal{F}(p) := \text{Eff}_K \{ f(p, x) \mid x \in X \} \tag{1.3}$$

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