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Asymptotic properties in parabolic problems dominated by a p-Laplacian operator with localized large diffusion

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ABSTRACT

This paper is concerned with upper semicontinuity of the family of attractors associated with nonlinear reaction–diffusion equations with principal part governed by a degenerate p-Laplacian in which the diffusion d_λ blows up in localized regions inside the domain.

1. Introduction

Let $\Omega \subset \mathbb{R}^n$, $n \geqslant 1$, be a smooth bounded domain with smooth boundary $\Gamma = \partial\Omega$, and $\lambda \in (0, 1]$ a parameter. In this work we study the asymptotic behavior of the solutions of the family of parabolic equations

$$\begin{cases} u_t^\lambda - \operatorname{div}(d_\lambda(x)|\nabla u^\lambda|^{p-2}\nabla u^\lambda) + |u^\lambda|^{p-2}u^\lambda = B(u^\lambda) & \text{in } \Omega \\ u^\lambda = 0 & \text{on } \Gamma, \\ u^\lambda(0) = u_0^\lambda, \end{cases} \tag{1.1}$$

as $\lambda \to 0$. The parameter λ represents the fact that, as $\lambda \to 0$, the diffusion d_λ is going to infinity in a localized region Ω_0 inside the physical domain Ω. We assume that $p > 2$ and that B is globally Lipschitz and uniformly integrable.

Next we introduce some notation following [1]. Let Ω_0 be a smooth subdomain of Ω, with $\bar{\Omega}_0 \subset \Omega$, $\Omega_0 = \bigcup_{i=1}^m \Omega_{0,i}$, where m is a positive integer and $\Omega_{0,i}$ are connected smooth subdomains of Ω with $\bar{\Omega}_{0,i} \cap \bar{\Omega}_{0,j} = \varnothing$, for $i \neq j$. Define $\Omega_1 = \Omega \setminus \bar{\Omega}_0$, and $\Gamma_{0,i} = \partial\Omega_{0,i}$, $\Gamma_0 = \bigcup_{i=1}^m \Gamma_{0,i}$ as the boundaries of $\Omega_{0,i}$ and Ω_0, respectively. Notice that $\partial\Omega_1 = \Gamma \cup \Gamma_0$.

The diffusion coefficients $d_\lambda : \Omega \subset \mathbb{R}^n \to \mathbb{R}$ are bounded and smooth functions in Ω, satisfying

$$0 < m_0 \leqslant d_\lambda(x) \leqslant M_\lambda, \tag{1.2}$$

for all $x \in \Omega$ and $0 < \lambda \leqslant 1$. We also assume that the diffusion is large in Ω_0 as $\lambda \to 0$, or more precisely,

$$d_\lambda(x) \to \begin{cases} d_0(x), & \text{uniformly on } \Omega_1, \ (d_0 \in \mathcal{C}^1(\bar{\Omega}_1, (0, \infty))); \\ \infty, & \text{uniformly on compact subsets of } \Omega_0. \end{cases}$$

It is important to notice here that the assumption that $\Gamma \cap \Gamma_0 = \varnothing$, that is, the diffusion is large in the interior of Ω, is crucial in the development of our analysis.

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