Nonlinear Analysis 74 (2011) 4503–4510

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Conley index condition for asymptotic stability[☆]

Emmanuel Moulay[a,*], Qing Hui[b,1]

[a] Xlim (UMR-CNRS 6172), Département SIC, Université de Poitiers, Bât. SP2MI, Bvd Marie et Pierre Curie, BP 30179, 86962 Futuroscope Chasseneuil Cedex, France

[b] Department of Mechanical Engineering, Texas Tech University, Lubbock, TX 79409-1021, USA

ARTICLE INFO

Article history:
Received 16 September 2010
Accepted 2 April 2011
Communicated by V. Lakshmikantham

Keywords:
Asymptotic Lyapunov stability
Conley index

ABSTRACT

In this paper, we use Conley index theory to develop necessary conditions for stability of equilibrium and periodic solutions of nonlinear continuous-time systems. The Conley index is a topological generalization of the Morse theory which has been developed to analyze dynamical systems using topological methods. In particular, the Conley index of an invariant set with respect to a dynamical system is defined as the relative homology of an index pair for the invariant set. The Conley index can then be used to examine the structure of the system invariant set as well as the system dynamics within the invariant set, including system stability properties. Efficient numerical algorithms using homology theory have been developed in the literature to compute the Conley index and can be used to deduce the stability properties of nonlinear dynamical systems.

1. Introduction

One of the most basic issues in system theory is the stability of dynamical systems. The most complete contribution to stability analysis of nonlinear dynamical systems was introduced in the late nineteenth century by Lyapunov in his seminal work in 1892 entitled *The General Problem of the Stability of Motion* [1,2]. Lyapunov's results which include the direct and indirect methods, along with the Krasovskii–LaSalle invariance principle [3,4], provide a powerful framework for analyzing the stability of equilibrium and periodic solutions of nonlinear dynamical systems. Lyapunov's direct method for examining the stability of an equilibrium state of a dynamical system requires the construction of a positive-definite function of the system states (Lyapunov function) for which its time rate of change due to perturbations in a neighborhood of the system's equilibrium is always negative or zero. Stability of periodic solutions of a dynamical system can also be addressed by constructing a Lyapunov-like function satisfying the Krasovskii–LaSalle invariance principle.

Alternatively, in the case where the trajectory of a dynamical system can be relatively easily integrated, the Poincaré theorem [5] provides a powerful tool in analyzing the stability properties of periodic orbits and limit cycles. Specifically, the Poincaré theorem provides necessary and sufficient conditions for the stability of periodic orbits based on the stability properties of a fixed point of a discrete-time dynamical system constructed from a Poincaré return map. However, in many applications, especially for high-dimensional nonlinear systems, system trajectories cannot be relatively easily integrated and the construction of a Lyapunov function for establishing stability properties of a dynamical system can be a daunting task.

In this paper, we use Conley index theory [6–8] to develop necessary conditions for the stability of equilibrium and periodic solutions of nonlinear continuous-time systems. The Conley index is a topological generalization of the Morse

☆ The second author's work was supported by the Defense Threat Reduction Agency, Basic Research Award #HDTRA1-10-1-0090, to Texas Tech University.

* Corresponding author. Tel.: +33 5 49 49 68 55.
E-mail addresses: emmanuel.moulay@univ-poitiers.fr (E. Moulay), qing.hui@ttu.edu (Q. Hui).

1 Tel.: +1 806 742 3563 238.

0362-546X/$ – see front matter
doi:10.1016/j.na.2011.04.014