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# On sufficiency and duality for nonsmooth multiobjective programming problems involving generalized $V$–$r$-invex functions

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ABSTRACT

In this paper, a class of nonsmooth multiobjective programming problems with inequality constraints is considered. We introduce the concepts of $V$–$r$-pseudo-invex, strictly $V$–$r$-pseudo-invex and $V$–$r$-quasi-invex functions, in which the involved functions are locally Lipschitz. Based upon these generalized $V$–$r$-invex functions, sufficient optimality conditions for a feasible point to be an efficient or a weakly efficient solution are derived. Appropriate duality theorems are proved for a Mond–Weir-type dual program of a nonsmooth multiobjective programming under the aforesaid functions.



## 1. Introduction

In most real life problems, decisions are made taking into account several conflicting criteria, rather than by optimizing a single objective. Such a problem is called multiobjective programming or vector optimization. These problems arise in economics, human decision making, optimization, control engineering, transportation and many other diverse fields. For more applications, we refer the reader to [1–3].

Multiobjective programming has grown remarkably in different directions in the setting of optimality conditions and duality theory in the last three decades. It has been enriched by the applications of various types of generalizations of convexity theory, with and without differentiability assumptions (see for example [4–16], and others).

The class of invex functions was introduced by Hanson [17] for a differentiable function. Reiland [18] extended invexity to the non-differentiable setting by defining invexity for Lipschitz real-valued functions. His principle was the analytic tool of the generalized gradient of Clarke [19]. Kaul et al. [20] established optimality and duality results for non-differentiable programming problems involving Lipschitz functions under generalized invexity assumptions.

Jeyakumar and Mond [21] introduced a new class of functions namely $V$-invex functions and established sufficient optimality criteria and duality results in the multiobjective static case for weak minima solutions. Antczak [8] introduced the concept of $V$–$r$-invexity for differentiable multiobjective programming problems, which is a generalization of $V$-invex functions [21] and $r$-invex functions [7]. Kuk et al. [22] defined the concept of $V$–$\rho$-invexity for vector-valued functions, which is a generalization of the $V$-invex function [21] and proved the generalized Karush–Kuhn–Tucker sufficient optimality

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