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Regularity criteria for a Lagrangian-averaged magnetohydrodynamic-α model

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ABSTRACT

In this paper, we study the n-dimensional Lagrangian-averaged magnetohydrodynamic-α (LAMHD-α) model with $5 \leq n \leq 7$ throughout the whole space. Various regularity criteria are established for both the cases with and those without viscosity.

1. Introduction

The magnetohydrodynamics (MHD) equations are a well-known model in plasma physics, for describing the interactions between a magnetic field and a fluid made of moving electrically charged particles. The system has the form

$$\begin{aligned}
&\partial_t v + (v \cdot \nabla)v - \epsilon \Delta v + \nabla \pi + \frac{1}{2}\nabla |B|^2 = (B \cdot \nabla)B, \\
&\partial_t B + (v \cdot \nabla)B - (B \cdot \nabla)v - \Delta B = 0, \\
&\operatorname{div} v = \operatorname{div} B = 0, \quad x \in \mathbb{R}^n.
\end{aligned} \tag{1.1}$$

Here the unknowns are the velocity field v of the fluid, the pressure π and the magnetic field B. $\epsilon > 0$ is the constant kinematic viscosity.

When $B \equiv 0$, the MHD equations reduce to the classical Navier–Stokes equations. For the 3D Navier–Stokes equations, it is proved that the weak solution remains smooth in $(0, T) \times \mathbb{R}^3$ if the velocity v satisfies one of the following conditions [1–5]:

$$v \in L^r(0, T; L^p) \quad \text{with } \frac{2}{r} + \frac{3}{p} = 1, 3 \leq p \leq \infty, \tag{1.2}$$

$$\nabla v \in L^r(0, T; L^p) \quad \text{with } \frac{2}{r} + \frac{3}{p} = 2, 3/2 < p \leq \infty, \tag{1.3}$$

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