Nonlinear Analysis 74 (2011) 1190–1199

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Geometrical coefficients and the structure of the fixed-point set of asymptotically regular mappings in Banach spaces

Jarosław Górnicki

Department of Mathematics, Rzeszów University of Technology, P.O. Box 85, 35-959 Rzeszów, Poland

ARTICLE INFO

Article history:
Received 13 March 2010
Accepted 28 September 2010

MSC:
primary 47H09
47H10
secondary 47B20
54C15

Keywords:
Asymptotically regular mapping
Retract
Asymptotic center
Fixed point
Uniformly convex Banach space
Opial's property
Opial's modulus
Weakly convergent sequence coefficient

ABSTRACT

It is shown that if E is a separable and uniformly convex Banach space with Opial's property and C is a nonempty bounded closed convex subset of E, then for some asymptotically regular self-mappings of C the set of fixed points is not only connected but even a retract of C. Our results qualitatively complement, in the case of a uniformly convex Banach space, a corresponding result presented in [T. Domínguez, M.A. Japón, G. López, Metric fixed point results concerning measures of noncompactness mappings, in: W.A. Kirk, B. Sims (Eds.), Handbook of Metric Fixed Point Theory, Kluwer Acad. Publishers, Dordrecht, 2001, pp. 239–268].

1. Introduction

Asymptotic regularity is a fundamentally important concept in metric fixed-point theory (see [1–4] and the references therein). It was formally introduced by Browder and Petryshyn in 1966 [5].

Definition 1. Let (M, d) be a metric space. A mapping $T : M \to M$ is called *asymptotically regular* if $\lim_{n\to\infty} d(T^n x, T^{n+1}x) = 0$ for all $x \in M$.

Example 2. Let $T : [0, 1] \to [0, 1]$ be an arbitrary nonexpansive mapping. It is easy to check that $S = \frac{1}{2}(I + T)$ is also nonexpansive. Thus

$$|S^{n+1}x - S^n x| \leqslant \cdots \leqslant |S^2 x - Sx| \leqslant |Sx - x|.$$

Furthermore, S is a nondecreasing function. Indeed, if $x \leqslant y$ and $Sx > Sy$ we have $\frac{1}{2}(x + Tx) > \frac{1}{2}(y + Ty)$ which implies

$$|Tx - Ty| \geqslant Tx - Ty > y - x = |x - y|.$$

E-mail addresses: gornicki@prz.edu.pl, gornicki@dnet.pl.

0362-546X/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2010.09.059