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Almost automorphic solutions to abstract Volterra equations on the line

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ABSTRACT

Given $a \in L^1(\mathbb{R})$ and A the generator of an L^1-integrable family of bounded and linear operators defined on a Banach space X, we prove the existence of an almost automorphic mild solution to the semilinear integral equation $u(t) = \int_{-\infty}^{t} a(t-s)[Au(s)+f(s,u(s))]\mathrm{d}s$ for each $f : \mathbb{R} \times X \to X$ S^p-almost automorphic in t, uniformly in $x \in X$, and satisfying diverse Lipschitz type conditions. For the scalar linear case, we prove that $a \in L^1(\mathbb{R})$ completely monotonic is already sufficient.

1. Introduction

We study almost automorphic solutions of an integral equation with infinite delay in a general Banach space X:

$$u(t) = \int_{-\infty}^{t} a(t-s)[Au(s)+f(s,u(s))]\mathrm{d}s, \quad t \in \mathbb{R} \tag{1.1}$$

where the operator $A : D(A) \subset X \to X$ generates an integral resolvent and $a : \mathbb{R}_+ \to \mathbb{C}$ is an integrable function.

A rich source of problems leading to Eq. (1.1) is provided by the theory of viscoelastic material behavior. Some typical examples are provided by viscoelastic fluids and heat flow in materials of fading memory type: see for instance [1–3]. In such applications the operator A typically is the Laplacian in $X = L^2(\Omega)$ or the elasticity operator, the Stokes operator or biharmonic Δ^2, etc., equipped with suitable boundary conditions. The material kernel $a(t)$ reflects the properties of the medium under consideration. Note that, in the finite dimensional case, the system (1.1) contains as particular cases several systems with finite or infinite delay already considered in the literature. See e.g. [4,5].

An equivalent form of Eq. (1.1) is given by

$$u(t) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\alpha u(t) + \int_{-\infty}^{t} k(t-s)u(s)\mathrm{d}s\right) = \int_0^\infty a(s)\mathrm{d}s(Au(t)+f(t,u(t))), \quad t \in \mathbb{R} \tag{1.2}$$

for some $\alpha > 0$ and $k \in L^1(\mathbb{R}_+)$ nonnegative and nonincreasing; see [6, Section 2]. This integro-differential equation was studied in [7], where some results of [8] were used in order to obtain the existence and regularity of the solution u when A generates a contraction semigroup (not necessarily analytic) on X.

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