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Fixed point theorems for generalized weakly contractive mappings

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ABSTRACT

We prove a fixed point theorem for a generalized weakly contractive mapping and a fixed point theorem for a pair of weakly contractive mappings. We also show that these mappings satisfy properties P and Q.

1. Introduction

Until 1968 Banach's contraction principle was the main tool (and perhaps the only one) used to establish the existence and uniqueness of fixed points. It has been used in many different fields of mathematics, but suffers from one drawback. In order to use the contractive condition, a self-mapping T must be Lipschitz continuous, with Lipschitz constant $L < 1$. In particular, T must be continuous at all points of its domain. In 1968 Kannan [1] constructed a contractive condition which, like that of Banach, possessed a unique fixed point, which could be obtained by starting at any point x_0 in the space, and using function iteration defined by $x_{n+1} = Tx_n$ (also called Picard iteration). However, unlike the Banach condition, there exist discontinuous functions satisfying the definition of Kannan, although such mappings are continuous at the fixed point. Following the appearance of [1] many persons created contractive conditions not requiring continuity of the mapping. Today fixed point literature of contractive mappings contains over 500 such papers. One survey of a number of these conditions appears in [2]. Weak contraction principle is a generalization of Banach's contraction principle which was first given by Alber et al. in Hilbert spaces [3] and subsequently extended to metric spaces by Rhoades [4]. Fixed point problems involving weak contractions and mappings satisfying weak contractive type inequalities have been considered in several works like [5–9].

'Khan et al. [10] initiated the use of a control function in metric fixed point theory, which they called an altering distance function'. This function and its generalizations have been used in fixed point problems in metric and probabilistic metric spaces in works like [11–15].

The purpose of the work is to prove some fixed point results for generalized weakly contractive mappings by using a control function. Precisely, we prove two unique fixed point theorems. We also establish P and Q properties for these mappings.

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