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Global well-posedness for the critical dissipative quasi-geostrophic equations in L^∞

Hantaek Bae

Center for Scientific Computation and Mathematical Modeling, University of Maryland, College Park, MD, 20783, United States

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ABSTRACT

In this paper, we study the critical dissipative quasi-geostrophic equations in scaling invariant spaces. We prove that there exists a global-in-time small solution for small initial data $\theta_0 \in L^\infty \cap \dot{H}^1$ such that $\mathscr{R}(\theta_0) \in L^\infty$, where $\mathscr{R}$ is the Riesz transform. As a corollary, we prove that if in addition, $\theta_0 \in \dot{B}^0_{\infty,q}$, $1 \le q < 2$, is small enough, then $\theta \in \tilde{L}^\infty_t \dot{B}^0_{\infty,q} \cap \tilde{L}^1_t \dot{B}^1_{\infty,q}$.

1. Introduction

In this paper, we are concerned with the dissipative quasi-geostrophic equations in two dimensions. These equations are derived from the more general quasi-geostrophic approximation for nonhomogeneous fluid flow in a rapidly rotating three-dimensional half-space with small Rossby and Ekman numbers. The system of equations in two dimensions is given by

$$(\mathrm{DQG})_\alpha \begin{cases} \theta_t + v \cdot \nabla\theta + \kappa(-\triangle)^\alpha \theta = 0, \\ v = (-\mathscr{R}_2\theta, \mathscr{R}_1\theta), \end{cases}$$

where the scalar function θ represents the potential temperature, v is the fluid velocity, and $0 \le \alpha \le 1$. $(-\triangle)^\alpha$ is a pseudo-differential operator, which is denoted by $\Lambda^{2\alpha}$, such that $\mathscr{F}(\Lambda^{2\alpha} f) = |\xi|^{2\alpha}\mathscr{F}(f)$. Here, $\mathscr{R} = (\mathscr{R}_1, \mathscr{R}_2)$ are the usual Riesz transforms:

$$\mathscr{R}_l f(x) = \mathscr{F}^{-1}\left(\frac{i\xi_l}{|\xi|}\hat{f}(\xi)\right)(x), \quad l = 1, 2. \tag{1.1}$$

For simplicity, we take $\kappa = 1$. The cases $\alpha > \frac{1}{2}$, $\alpha = \frac{1}{2}$, and $\alpha < \frac{1}{2}$ are called respectively sub-critical, critical and super-critical.

The critical quasi-geostrophic equations are the dimensionally correct analogue to the three-dimensional Navier–Stokes equations. In two dimensions, the Navier–Stokes equations are globally well-posed, while the regularity problems for the three-dimensional Navier–Stokes equations are still open. However, the regularity problems for the critical quasi-geostrophic equations were solved recently by two groups. Caffarelli and Vasseur [1] proved the global Hölder regularity for the critical quasi-geostrophic equations. They used harmonic extension to prove a gain of regularity of weak solutions. Kiselev, Nazarov and Volberg [2] also proved that the critical quasi-geostrophic equation with periodic smooth initial data θ_0 has a unique global smooth solution, by using the modulus of continuity argument.

E-mail address: hbae@cscamm.umd.edu.

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