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Critical exponents and critical dimensions for quasilinear elliptic problems

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ABSTRACT

The main purpose of this paper is to discuss the critical dimension phenomenon for sign-changing solutions of the following quasilinear elliptic problem involving critical Sobolev exponent:

$$\begin{cases} -\Delta_p u = |u|^{p^*-2}u + \lambda |u|^{q-2}u, & x \in B_1, \\ u|_{\partial B_1} = 0, \end{cases}$$

where $B_1 \subset \mathbb{R}^N$ is a unit ball centered at the origin, $\Delta_p u = div(|\nabla u|^{p-2}\nabla u)$, $\lambda > 0$, $2 \le p < N$, $p \le q < p^*$, $p^* = \frac{Np}{N-p}$ is the critical Sobolev exponent for the embedding $W_0^{1,p}(B_1) \hookrightarrow L^{p^*}(B_1)$. We show that the above problem exists infinitely many sign-changing radial solutions if the space dimension $N > \frac{p(pq-q+1)}{1+(q-p)(p-1)}$.

1. Introduction and main results

It is well known from the work of Brezis and Nirenberg [1] that the existence of positive solutions of semilinear elliptic equations involving critical exponents relate to the dimension of space. More specially, for the representative problem

$$\begin{cases} -\Delta u = \lambda u + |u|^{2^*-2}u & \text{in } \Omega, \\ u|_{\partial\Omega} = 0, \end{cases} \tag{1.1}$$

where Ω is a bounded smooth open subset of $\mathbb{R}^N$, $N \ge 3$ and $2^* = \frac{2N}{N-2}$ is the critical exponent for Sobolev embedding. The following results were proved in [1].

(i) If $N \ge 4$, problem (1.1) has at least one positive solution $u \in H_0^1(\Omega)$ when $0 < \lambda < \lambda_1$.
(ii) If $N = 3$, problem (1.1) has at least one positive solution $u \in H_0^1(\Omega)$ when $\lambda_* < \lambda < \lambda_1$, where λ_* is a positive constant.
(iii) If $N = 3$ and Ω is a ball, then $\lambda_* = \frac{1}{4}\lambda_1$, and problem (1.1) has no positive solution for $\lambda \le \lambda_*$,

where λ_1 is the first eigenvalue of the operator $-\Delta$ in Ω with Dirichlet boundary condition.

The preceding results show that the space dimension N plays a fundamental role when people seeks positive solutions of (1.1), in particular, the dimension $N = 3$ is a special one, if compared with $N \ge 4$. According to the definition introduced by Pucci and Serrin (see [2], also see [3]), we shall say that $N = 3$ is a critical dimension for problem (1.1). In the celebrated paper [2,3], a wide class of nonlinear critical elliptic problems which exhibit the phenomenon of critical dimensions have been studied.

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