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Existence of homoclinic orbits for second order Hamiltonian systems without (AR) condition[☆]

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ABSTRACT

The existence of homoclinic orbits is obtained for a class of the second order Hamiltonian systems $\ddot{u}(t) - L(t)u(t) + \nabla W(t, u(t)) = 0$, $\forall t \in R$, by the local linking lemma, where $W(t, x)$ is superquadratic and need not be nonnegative globally.

1. Introduction and main results

Let us consider the second order Hamiltonian systems

$$\ddot{u}(t) - L(t)u(t) + \nabla W(t, u(t)) = 0, \quad \forall t \in R, \tag{1}$$

where $L \in C(R, R^{N^2})$ is a symmetric matrix valued function, $W \in C^1(R \times R^N, R)$ and $\nabla W(t, x) = \frac{\partial W}{\partial x}(t, x)$. We say that a solution u of problem (1) is homoclinic (to 0) if $u \in C^2(R, R^N)u(t) \to 0$ as $|t| \to \infty$ and $\dot{u}(t) \to 0$ as $|t| \to \infty$. Homoclinic orbits of dynamical systems are important in applications. From their existence and under certain conditions, one may infer the existence of chaos nearby or the bifurcation behavior of periodic orbits.

With the variational methods, the existence and multiplicity of homoclinic orbits of problem (1) have been obtained by many papers (see [1–19]). Since the domain is unbounded, there is a lack of compactness of the Sobolev embedding. Many papers consider the periodic (autonomous, asymptotically periodic) problems (see [1–3,5,6,15,16]). Some papers treat the symmetric case (see [8,12]). Recently the coercive case has been dealt with, that is,

the smallest eigenvalue of $L(t) \to +\infty$ as $|t| \to \infty$

(see [4,13,14,16–19]). In most superquadratic cases, there is a so-called global (AR) condition on W, that is, there exists a constant $\mu > 2$ such that

$$0 < \mu W(t, x) \le (\nabla W(t, x), x) \tag{2}$$

for all $t \in R$ and $x \in R^N \setminus \{0\}$, which is very important to guarantee the boundedness of the $(PS)_c$ sequence (see [1–4,6, 15,16]). Obviously, W in (2) is asked to be positive globally. In [14], the authors study the homoclinic orbits of problem (1) without (2). However, under conditions in [14], W is still asked to be nonnegative globally. In this paper, we use the local linking lemmas introduced in [10,11] to remove this unnecessary condition. The main results are the following theorems.

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