



Convergence theorems for nonexpansive semigroups in CAT(0) spaces [☆]

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ABSTRACT

Let C be a convex and compact subset of a CAT(0) space X . In this paper, we consider the following iterative scheme for a one-parameter nonexpansive semigroup $\{T(t) : t \geq 0\}$ on C :

$$\begin{cases} x_1 \in C, \\ x_{n+1} = \lambda T(t_n)x_n \oplus (1 - \lambda)x_n, \end{cases}$$

where $\lambda \in (0, 1)$ and $\{t_n\} \subset [0, \infty)$, and we prove that, under certain conditions, $\{x_n\}$ converges to a common fixed point of the semigroup $\{T(t) : t \geq 0\}$.

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1. Introduction and preliminaries

We denote the set of all positive integers by \mathbb{N} and the set of all real numbers by \mathbb{R} . Let (X, d) be a metric space. A mapping $T : X \rightarrow X$ is said to be *nonexpansive* if

$$d(Tx, Ty) \leq d(x, y)$$

for all $x, y \in X$. We denote the set of all fixed points of T by $F(T)$, i.e.,

$$F(T) = \{x \in X : Tx = x\}.$$

Let C be a subset of X and, for each $n \in \mathbb{N}$, $T_n : C \rightarrow X$ be mappings with $\bigcap_{n=1}^{\infty} F(T_n) \neq \emptyset$. The family $\{T_n\}$ is said to be *uniformly asymptotically regular* if, for each bounded subset B of C ,

$$\lim_{n \rightarrow \infty} \sup_{z \in B} d(T_n z, T_i(T_n z)) = 0$$

for all $i \in \mathbb{N}$.

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