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Algorithms with strong convergence for a system of nonlinear variational inequalities in Banach spaces

Yonghong Yao[a], Yeong-Cheng Liou[b], Shin Min Kang[c,*], Youli Yu[d]

[a] Department of Mathematics, Tianjin Polytechnic University, Tianjin 300387, China
[b] Department of Information Management, Cheng Shiu University, Kaohsiung 833, Taiwan
[c] Department of Mathematics and the RINS, Gyeongsang National University, Jinju 660-701, Republic of Korea
[d] School of Mathematics and Information Engineering, Taizhou University, Linhai 317000, China

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ABSTRACT

In this paper, a general system of nonlinear variational inequality problem in Banach spaces was considered, which includes some existing problems as special cases. For solving this nonlinear variational inequality problem, we construct two methods which were inspired and motivated by Korpelevich's extragradient method. Furthermore, we prove that the suggested algorithms converge strongly to some solutions of the studied variational inequality.

1. Introduction

In this paper, we are concerned with a general system of nonlinear variational inequality in Banach spaces (GSVIB), which involves finding $(x^*, y^*) \in C \times C$ such that

$$\begin{cases} \langle \lambda Ay^* + x^* - y^*, j(x - x^*)\rangle \geq 0, & \forall x \in C, \\ \langle \mu Bx^* + y^* - x^*, j(x - y^*)\rangle \geq 0, & \forall x \in C, \end{cases} \tag{1.1}$$

where X is a real Banach space, $C \subset X$ is a nonempty, closed and convex set, $A, B : C \to X$ are two nonlinear mappings and λ and μ are two positive real numbers.

Special cases

(I) If X is a real Hilbert space, then (1.1) reduces to

$$\begin{cases} \langle \lambda Ay^* + x^* - y^*, x - x^*\rangle \geq 0, & \forall x \in C, \\ \langle \mu Bx^* + y^* - x^*, x - y^*\rangle \geq 0, & \forall x \in C, \end{cases} \tag{1.2}$$

* Corresponding author. Tel.: +82 55 755 1917; fax: +82 55 755 1917.
E-mail addresses: yaoyonghong@yahoo.cn (Y. Yao), simplex_liou@hotmail.com (Y.-C. Liou), smkang@gnu.ac.kr (S.M. Kang), yuyouli@tzc.edu.cn (Y. Yu).

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