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Limit behavior of the global solutions to the Degasperis–Procesi-type equation

Fei Guo

School of Mathematical Sciences and Jiangsu Key Laboratory for NSLSCS, Nanjing Normal University, Nanjing 210046, PR China

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ABSTRACT

In this paper, we study the Degasperis–Procesi equation with a physically perturbation term—a linear dispersion. Based on the global existence result, we show that the solution of the Degasperis–Procesi equation with linear dispersion tends to the solution of the corresponding Degasperis–Procesi equation as the dispersive parameter goes to zero. Moreover, we prove that smooth solutions of the equation have finite propagation speed: they will have compact support if its initial data has this property.

1. Introduction

This paper is a sequel to [1,2]. We study the following Degasperis–Procesi equation with a linear dispersion term [3,4]

$$y_t + uy_x + 3u_x y + \gamma y_x = 0, \quad t > 0, x \in \mathbb{R}, \tag{1.1}$$

with $y = u - u_{xx}$ and $\gamma \in \mathbb{R}$. Applying a reciprocal transformation to Eq. (1.1), Degasperis, Holm and Hone [3] used the Painlevé analysis to show the formal integrability of Eq. (1.1) as Hamiltonian systems by constructing a Lax pair and a bi-Hamiltonian structure.

Indeed, the isospectral problem in the Lax pair for the DP equation is of third-order instead of second [3], and consequently is not self-adjoint,

$$\psi_x - \psi_{xxx} - \lambda y \psi = 0,$$

and

$$\psi_t + \frac{1}{\lambda}\psi_{xx} + (u+\gamma)\psi_x - \left(u_x + \frac{2}{3\lambda}\right)\psi = 0.$$

Note that when $\gamma = 0$, Eq. (1.1) is the well-known Degasperis–Procesi (DP) equation, i.e.,

$$y_t + uy_x + 3u_x y = 0, \quad t > 0, \ x \in \mathbb{R}. \tag{1.2}$$

Another closed relevant equation in the shallow water regime is the Camassa–Holm (CH) equation

$$y_t + uy_x + 2u_x y = 0, \quad t > 0, \ x \in \mathbb{R}, \tag{1.3}$$

where both solutions $u(t, x)$ of these two equations are considered as the horizontal component of the fluid velocity at time t in the spatial x-direction with momentum density $y = u - u_{xx}$, but evaluated at the different level line of the fluid

E-mail address: guof@njnu.edu.cn.

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