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Fixed point theorems for mixed monotone operators and applications to integral equations

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The purpose of this paper is to present some coupled fixed point theorems for a mixed monotone operator in a complete metric space endowed with a partial order by using altering distance functions. We also present an application to integral equations.

1. Introduction and background

Mixed monotone operators were introduce by Guo and Lakshmikantham in [1]. Their study has not only important theoretical meaning but also wide applications in engineering, nuclear physics, biological chemistry technology, etc. (see [1–6]).

The purpose of this paper is to present some coupled fixed point theorems for a mixed monotone operator in the context of ordered metric spaces involving altering distance functions. These theorems are generalizations of the results of Bhaskar and Lakshmikantham [7]. Moreover, we present an application to integral equations.

Existence of fixed points in partially ordered sets has been considered recently in [8–22]. Tarski's theorem is used in [15] to show the existence of solutions for fuzzy equations and in [17] to prove existence theorems for fuzzy differential equations. In [12–16,19] some applications to ordinary differential equations and in [22] ones to matrix equations are presented. In [7,9,14,23] some fixed point theorems are proved for a mixed monotone mapping in a metric space endowed with a partial order and the authors apply their results to problems of existence and uniqueness of solutions for some boundary value problems.

In the context of ordered metric spaces the usual contraction is weakened but at the expense of the operator being monotone. The main idea in [16,22] involves combining the ideas of the contraction principle with those of the monotone iterative technique [24].

Now we briefly recall various basic definitions and facts.

Let $(X, \leq)$ be a partially ordered set and $F : X \times X \longrightarrow X$. We say that F has the mixed monotone property if $F(x, y)$ is monotone nondecreasing in x and is monotone nonincreasing in y, that is, for any $x, y \in X$,

$$x_1, x_2 \in X, \quad x_1 \leq x_2 \Rightarrow F(x_1, y) \leq F(x_2, y)$$

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