Nonlinear Analysis 74 (2011) 6781–6787

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Characterization of d.c. functions in terms of quasidifferentials

Ivan Ginchev [a,b,*], Juan-Enrique Martínez-Legaz [c]

[a] University of Insubria, Faculty of Economics, Via Monte Generoso 71, 21100 Varese, Italy

[b] Technical University of Varna, Department of Mathematics, Studentska Str. 1, 9010 Varna, Bulgaria

[c] Universitat Autònoma de Barcelona, Departament d'Economia i d'Història Econòmica, 08193 Bellaterra, Spain

ARTICLE INFO

Article history:
Received 10 January 2011
Accepted 2 July 2011
Communicated by Ravi Agarwal

MSC:
26B25
49J52

Keywords:
Generalized convexity
d.c. functions
Quasidifferentials
Characterization of d.c. functions

ABSTRACT

A characterization of d.c. functions $f : \Omega \to \mathbb{R}$ in terms of the quasidifferentials of f is obtained, where Ω is an open convex set in a real Banach space. Recall that f is called d.c. (difference of convex) if it can be represented as a difference of two finite convex functions. The relation of the obtained results with known characterizations is discussed, specifically the ones from [R. Ellaia, A. Hassouni, Characterization of nonsmooth functions through their generalized gradients, Optimization 22 (1991), 401–416] in the finite-dimensional case and [A. Elhilali Alaoui, Caractérisation des fonctions DC, Ann. Sci. Math. Québec 20 (1996), 1–13] in the case of a Banach space.

1. Introduction

In this paper, unless otherwise specified, X stands for a real Banach space with norm $\|\cdot\|$ and Ω is a nonempty open convex subset of X. The topological dual of X is denoted by X^* and by $\langle\cdot,\cdot\rangle$, the canonical dual pairing. In the finite-dimensional case, we consider occasionally $\mathbb{R}^n$ as the Euclidean space with the canonical scalar product and identify $\mathbb{R}^n$ with its dual.

Definition 1.1. The function $f : \Omega \to \mathbb{R}$ is called d.c. (difference of convex) if it can be represented as a difference

$$f(x) = g(x) - h(x), \quad x \in \Omega, \tag{1.1}$$

of two convex functions g, h, finite on Ω.

Here it is convenient to consider g and h defined on X and having values in $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$ in order to use the usual conventions and results from convex analysis. Let us say that equality (1.1) does not depend on the way g and h are defined on the complement of Ω. We may take them, e.g. equal to $+\infty$ on $X \setminus \Omega$ (with say the convention $\infty - \infty = \infty$). In this definition, Ω can be an arbitrary convex set, not necessarily open, though in the present paper we deal with open convex sets only. Some authors define d.c. functions as functions admitting a decomposition of two convex continuous functions. The continuity of the decomposing functions is not assumed in our definition.

There are many reasons to study d.c. functions. The class $\mathrm{DC}(\Omega)$ of d.c. functions on Ω is the vector space generated by the class of convex functions on Ω. When Ω is compact, $\mathrm{DC}(\Omega)$ is dense in $C(\Omega)$ (the space of the continuous functions supplied with the sup norm) it includes some important classes as say the class of C^2 functions (in finite dimensions [1–3]), and obeys remarkable stability with respect to the operations usually used in optimization.

* Corresponding author at: University of Insubria, Faculty of Economics, Via Monte Generoso 71, 21100 Varese, Italy.
E-mail addresses: vechnig@yahoo.com (I. Ginchev), JuanEnrique.Martinez.Legaz@uab.cat (J.-E. Martínez-Legaz).

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doi:10.1016/j.na.2011.07.003