Nonlinear Analysis 74 (2011) 965–973

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Wave breaking for the periodic weakly dissipative Dullin–Gottwald–Holm equation

Zhengguang Guo [a,b,*], Lidiao Ni [a]

[a] Department of Mathematics, Zhejiang Normal University, Jinhua 321004, Zhejiang, China

[b] Department of Mathematics, East China Normal University, Shanghai 200241, China

ARTICLE INFO

Article history:
Received 6 July 2010
Accepted 23 September 2010

MSC:
37L05
35Q58
26A12
58E35

Keywords:
The Dullin–Gottwald–Holm equation
Wave breaking
Optimal constant

ABSTRACT

In this paper, we consider the periodic weakly dissipative Dullin–Gottwald–Holm equation. The present work is mainly concerned with blow-up phenomena for the Cauchy problem for this new kind of equation. We apply the optimal constant to give sufficient conditions via an appropriate integral form of the initial data, which guarantee the finite-time singularity formation for the corresponding solution.

1. Introduction

In this paper, we consider the following DGH (Dullin–Gottwald–Holm) equation with weakly dissipative term:

$$u_t - \alpha^2 u_{xxt} + c_0 u_x + 3uu_x + \gamma u_{xxx} + \lambda \left(1 - \alpha^2 \partial_x^2\right) u = \alpha^2 \left(2u_x u_{xx} + uu_{xxx}\right). \tag{1.1}$$

Here the constants α^2 and γ/c_0 are squares of length scales, and the constant $c_0 = \sqrt{gh} > 0$ is the critical shallow water speed for undisturbed water at rest at spatial infinity, where h is the mean fluid depth and g is the gravitational constant, $g = 9.8\ \mathrm{m/s^2}$. In addition, $\lambda \left(1 - \alpha^2 \partial_x^2\right) u$ is the weakly dissipative term, and $\lambda > 0$ is a constant.

Recall that Dullin et al. [1] derived a new equation describing the unidirectional propagation of surface waves in a shallow water regime:

$$u_t - \alpha^2 u_{xxt} + c_0 u_x + 3uu_x + \gamma u_{xxx} = \alpha^2 \left(2u_x u_{xx} + uu_{xxx}\right). \tag{1.2}$$

In what follows, we call this new integrable shallow water equation (1.2) the DGH equation.

When $\gamma = 0$ and $\alpha = 1$ in (1.2), we recover the shallow water (Camassa–Holm) equation derived physically by Camassa and Holm in [2] by directly approximating the Hamiltonian for Euler's equations in the shallow water regime, where $u(x, t)$ represents the free surface above a flat bottom. Moreover, the alternative physical derivation of the Camassa–Holm equation as a model for shallow water waves of moderate amplitude was obtained by Johnson [3]. The properties of the well-posedness, wave breaking criteria for a large class of initial data, global existence and propagation speed have been

* Corresponding author at: Department of Mathematics, Zhejiang Normal University, Jinhua 321004, Zhejiang, China.
E-mail addresses: gzgmath@gmail.com (Z. Guo), ni.lidiao@gmail.com (L. Ni).

0362-546X/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2010.09.050