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Periodic solutions of delay impulsive differential equations[☆]

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ABSTRACT

We study the following semilinear impulsive differential equation with delay:

$$\begin{aligned} &u'(t) + Au(t) = f(t, u(t), u_t), \quad t > 0,\ t \neq t_i, \\ &u(s) = \phi(s), \quad s \in [-r, 0], \\ &\Delta u(t_i) = I_i(u(t_i)), \quad i = 1, 2, \ldots,\ 0 < t_1 < t_2 < \cdots < \infty, \end{aligned}$$

in a Banach space $(X, \|\cdot\|)$ with an unbounded operator A, where $r > 0$ is a constant and $u_t(s) = u(t+s), s \in [-r, 0]$. Here, $\Delta u(t_i) = u(t_i^+) - u(t_i^-)$ constitutes an impulsive condition, which can be used to model more physical phenomena than the traditional initial value problems. We assume that $f(t, u, w)$ is T-periodic in t and then prove with some compactness conditions that if solutions of the equation are ultimately bounded, then the differential equation has a T-periodic solution. The new results obtained here extend some results in this area for differential equations without impulsive conditions or without delays.

1. Introduction

Let $(X, \|\cdot\|)$ be a Banach space and let A be an unbounded operator. A study of periodic solutions for the following finite delay differential equation:

$$\begin{aligned} &u'(t) + Au(t) = f(t, u(t), u_t), \quad t > 0, \\ &u(s) = \phi(s), \quad s \in [-r, 0], \end{aligned}$$

was given in [1] using the boundedness of the solutions; see [2–6] for some related references.

Recently, a study of periodic solutions for the following impulsive differential equation:

$$\begin{aligned} &u'(t) + Au(t) = f(t, u(t)), \quad t > 0,\ t \neq t_i, \\ &u(0) = u_0, \\ &\Delta u(t_i) = I_i(u(t_i)), \quad i = 1, 2, \ldots,\ 0 < t_1 < t_2 < \cdots < \infty, \end{aligned}$$

was given in [7] using the boundedness of the solutions, where $\Delta u(t_i) = u(t_i^+) - u(t_i^-)$ constitutes an impulsive condition. Note that impulsive conditions are combinations of the traditional initial value problems and the short-term perturbations

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