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Nagumo type existence results of Sturm–Liouville BVP for impulsive differential equations[☆]

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ABSTRACT

In this paper, the method of upper and lower solutions and the Schauder degree theory are employed in the study of Sturm–Liouville boundary value problems for second order impulsive differential equations. We obtain the existence of at least three solutions to the problem under the assumption that the nonlinear term f satisfies a Nagumo condition with respect to the first order derivative.

1. Introduction

Impulsive differential equations serve as basic models to study the dynamics of processes which are subject to sudden changes in their states. Recent development in this field has been motivated by many applied problems, such as population dynamics [1], medicine [2–4] and control theory [5,6]. For the general aspects of impulsive differential equations, we refer the reader to the classical monographs [7–9]. In recent years, boundary value problem (BVP) of nonlinear impulsive differential equations have received a considerable attention; see [10–20,9,21,22] and the references therein. The purpose of the present paper is to investigate the existence of solutions for the following Sturm–Liouville boundary value problem impulse effect

$$\begin{cases} -u''(t) = f(t, u(t), u'(t)), \quad t \in J, t \neq t_k, \\ u(t_k^+) = P_k(u(t_k)), \quad k = 1, 2, \ldots, m, \\ u'(t_k^+) = Q_k(u'(t_k)), \quad k = 1, 2, \ldots, m, \\ au(0) - bu'(0) = 0, \qquad cu(1) + du'(1) = 0; \end{cases} \tag{1.1}$$

where $a > 0, b \geq 0, c > 0, d \geq 0, \rho = ac + ad + bc > 0$. $0 = t_0 < t_1 < t_2 < \cdots < t_m < t_{m+1} = 1$. $J = [0, 1], J_0 = [0, t_1]$ and $J_k = (t_k, t_{k+1}]$ $(k = 1, 2, \ldots, m)$. $f \in C(J \times R^2, R)$, $P_k, Q_k : R \to R$ are continuous. $u'(t_k^+), u(t_k^+)(u'(t_k), u(t_k))$ denote the right limit (left limit) of $u'(t)$ and $u(t)$ at $t = t_k$, respectively.

In [23], Erbe and Wang considered the following boundary value problems

$$\begin{cases} u''(t) + a(t)f(u) = 0, \quad 0 < t < 1, \\ au(0) - bu'(0) = 0, \qquad cu(1) + du'(1) = 0, \end{cases} \tag{1.2}$$

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