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Noncreasy and uniformly noncreasy Orlicz–Bochner function spaces[☆]

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ABSTRACT

We point out that uniform noncreasiness is a super-property in Banach spaces. Moreover, we prove that Orlicz–Bochner function spaces are noncreasy if and only if they are rotund or smooth. Finally, we obtain that Orlicz–Bochner function spaces are uniformly noncreasy if and only if they are uniformly rotund or uniformly smooth.

1. Introduction

Let X be a Banach space and X^* be its dual space. Denote by $B(X)$ and $S(X)$ the unit ball and the unit sphere of X, respectively. For $x \in S(X)$ define $\Sigma(x) = \{x^* \in S(X^*) : \langle x^*, x\rangle = 1\}$.

A Banach space X is said to have a crease if there exist $x^* \neq y^*$ in $S(X^*)$ such that $\operatorname{diam} S(x^*, y^*, 0) > 0$, where $S(x^*, y^*, \delta) = S(x^*, \delta) \cap S(y^*, \delta)$, $S(x^*, \delta) = \{x \in B(X) : x^*(x) \geq 1 - \delta\}$. Uniform rotundity and uniform smoothness can be easily characterized in terms of the slices $S(x^*, \delta)$ or $S(x^*, y^*, \delta)$ [1].

Remark 1 (*[1]*)**.** A Banach space X is uniformly rotund if and only if for every $\varepsilon > 0$ there exists $\delta > 0$ such that $\operatorname{diam} S(x^*, \delta) \leq \varepsilon$ for all functionals $x^* \in S(X^*)$.

Remark 2 (*[1]*)**.** A Banach space X is uniformly smooth if and only if for every $\varepsilon > 0$ there exists $\delta > 0$ such that $S(x^*, y^*, \delta) = \emptyset$ for all functionals $x^*, y^* \in S(X^*)$ with $\|x^* - y^*\| \geq \varepsilon$.

A Banach space X is said to be noncreasy (NC) if for all $x^* \neq y^*$ in $S(X^*)$, $\operatorname{diam} S(x^*, y^*, 0) = 0$. A Banach space X is said to be uniformly noncreasy (UNC) provided that for every $\varepsilon > 0$ there exists $\delta > 0$ such that if x^*, y^* in $S(X^*)$ and $\|x^* - y^*\| \geq \varepsilon$, then $\operatorname{diam} S(x^*, y^*, \delta) \leq \varepsilon$ [1].

UNC and NC Banach spaces were introduced by Prus [1]. He showed [1] that if X is uniformly rotund (smooth), then X is uniformly noncreasy, and uniformly noncreasy Banach spaces are reflexive; if X is rotund (smooth), then X is NC, and X is uniformly noncreasy if and only if X^* is uniformly noncreasy. Furthermore, he also proved that UNC spaces have the fixed point property, and showed examples of UNC spaces that fail to have normal structure. In this paper, we determine conditions for Orlicz–Bochner function spaces equipped with the Orlicz/Luxemburg norm to be UNC or NC.

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