



# Pseudo-almost automorphic mild solutions to semilinear integral equations in a Banach space

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## ABSTRACT

In this paper, we consider the existence of pseudo-almost automorphic solutions of the semilinear integral equation  $x(t) = \int_{-\infty}^t a(t-s)[Ax(s) + f(s, x(s))]ds$ ,  $t \in \mathbb{R}$  in a Banach space  $\mathbb{X}$ , where  $a \in L^1(\mathbb{R}_+)$ ,  $A$  is the generator of an integral resolvent family of linear bounded operators defined on the Banach space  $\mathbb{X}$ , and  $f : \mathbb{R} \times \mathbb{X} \rightarrow \mathbb{X}$  is a pseudo-almost automorphic function. The main results are proved by using integral resolvent families combined with the theory of fixed points.

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## 1. Introduction

In this paper, we are mainly concerned with the existence of pseudo-almost automorphic mild solutions to the class of semilinear integral equations in the abstract form

$$x(t) = \int_{-\infty}^t a(t-s)[Ax(s) + f(s, x(s))]ds, \quad t \in \mathbb{R}, \quad (1.1)$$

where  $a \in L^1(\mathbb{R}_+)$ ,  $A : D(A) \subseteq \mathbb{X} \rightarrow \mathbb{X}$  is the generator of an integral resolvent family defined on a complex Banach space  $\mathbb{X}$ , and  $f : \mathbb{R} \times \mathbb{X} \rightarrow \mathbb{X}$  is a pseudo-almost automorphic function satisfying some suitable conditions (see Section 3).

The concept of pseudo-almost automorphic functions suggested by N'Guérékata [1, page 40] was developed by Xiao et al. [2]. It is a natural generalization of both the classical almost automorphy in the sense of Bochner [3,4] and that of pseudo-almost periodicity due to Zhang [5–7]. Since then, these functions have generated lot of developments and applications. For more details we refer the reader to [1,8–10] and the references therein. The existence of almost automorphic and pseudo-almost automorphic solutions are among the most attractive topics in the qualitative theory of differential equations because of their significance and applications in physics, mechanics and mathematical biology. In recent years, the existence of automorphic and pseudo-almost automorphic solutions on different kinds of differential equations have been investigated in many publications such as [11–22] and the references therein.

Recently, Cuevas and Lizama [23] studied the existence and uniqueness of almost automorphic solutions for problem (1.1). In [24], the authors investigated the existence and regularity of compact almost automorphic solutions

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