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Fixed point theorems for random pseudo-contractive mappings

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ABSTRACT

Very recently in Fierro et al. (2009) [6], we obtained a general principle to prove the existence of Random Fixed Point Theorems. As a consequence of this, we have been able to obtain various generalizations for pseudo-contractive mappings with rather simple proofs. In addition, while we were deriving these extensions for random operators, some deterministic results arose, which also appear to be new.

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1. Introduction

Most recently, random fixed point theory has emerged as a very interesting area of research due to the close connection to random matrices, random partial differential equations and the various types of random operators arising in physical problems. In 1976, Bharucha-Reid [1] wrote a survey on the topic, revitalizing the interest in the area. As a consequence of this, many interesting results have emerged.

Let (X, d) be a metric space and S a closed and non-empty subset of X. Denote by 2^X (respectively $\mathcal{C}(X)$) the family of all non-empty (respectively non-empty and closed) subsets of X. A mapping $T : S \to 2^X$ is said to satisfy *condition* $(\mathcal{P})$ if for every closed ball B of S with radius $r \geq 0$ and any sequence $\{x_n\}$ in S for which $d(x_n, B) \to 0$ and $d(x_n, T(x_n)) \to 0$ as $n \to \infty$, there exists $x_0 \in B$ such that $x_0 \in T(x_0)$ where $d(x, B) = \inf\{d(x, y) : y \in B\}$. If Ω is any non-empty set, we say that the operator $T : \Omega \times S \to 2^X$ satisfies the condition $(\mathcal{P})$ if for each $\omega \in \Omega$, the mapping $T(\omega, \cdot) : S \to 2^X$ satisfies the condition $(\mathcal{P})$. We should observe that this latter condition is related to a condition that was originally introduced by Petryshyn [2] for single valued operators, to prove existence of fixed points. In our case, the condition is used to prove the measurability of the mapping *fixed point*, as a function of ω.

Let $(\Omega, \mathcal{A})$ be a measurable space and let (X, d) be a metric space. A mapping $F : \Omega \to 2^X$, is said to be measurable if $F^{-1}(G) = \{\omega \in \Omega : F(\omega) \cap G \neq \phi\}$ is measurable for open subset G of X. Notice that if X is separable and if for each closed subset C of X, the set $F^{-1}(C)$ is measurable, then F is measurable. Let C be a non-empty subset of X and $F : C \to 2^X$, then we say that F is lower (upper) semi-continuous if $F^{-1}(A)$ is open (closed) for all open (closed) subsets A of X. We say that F is continuous if F is, both, lower and upper semi-continuous.

A mapping $F : \Omega \times X \to Y$ is called a random operator if for each $x \in X$, the mapping $F(\cdot, x) : \Omega \to Y$ is measurable. Similarly a multi-valued mapping $F : \Omega \times X \to 2^Y$ is also called a random operator if for each $x \in X$, $F(\cdot, x) : \Omega \to 2^Y$ is measurable. A measurable mapping $\xi : \Omega \to Y$ is called a measurable selection of the operator $F : \Omega \to 2^Y$ if $\xi(\omega) \in F(\omega)$ for each $\omega \in \Omega$. A measurable mapping $\xi : \Omega \to X$ is called a random fixed point of the random operator $F : \Omega \times X \to X$ (or $F : \Omega \times X \to 2^X$) if for every $\omega \in \Omega$, $\xi(\omega) = F(\omega, \xi(\omega))$ (or $\xi(\omega) \in F(\omega, \xi(\omega))$). For the sake of clarity, we mention that $F(\omega, \xi(\omega)) = F(\omega, \cdot)(\xi(\omega))$.

Let C be a closed subset of the Banach space X and suppose F is a mapping from C into the topological vector space Y. We say the F is *demiclosed* at $y_0 \in Y$ if for any sequences $\{x_n\}$ in C and $\{y_n\}$ in Y with $y_n \in F(x_n)$, such that $\{x_n\}$ converges weakly

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