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Nonlinear Volterra integral equations and the Schröder functional equation

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ABSTRACT

We show an interesting connection between a special class of Volterra integral equations and the famous Schröder equation. The basic results provide criteria for the existence of nontrivial as well as blow-up solutions of the Volterra equation, expressed in terms of the convergence of some integrals. Examples related to Volterra equations with power and exponential nonlinearities are presented.

1. Introduction

In last 30 years many articles have been dedicated to second-kind nonlinear Volterra integral equations of a certain type with a convolution kernel, i.e.

$$u(t) = \int_0^t k(t-s)g(u(s))\mathrm{d}s, \quad t \geq 0, \tag{1}$$

where $g(0) = 0$. Eq. (1) appears in many physical problems, for example in the theory of water percolation from cylindrical reservoirs [1] and in the theory of nonlinear waves in tubes [2]. Just from an application point of view, the problem of the existence of nontrivial solutions and blow-up solutions is very interesting. Recall that by a nontrivial solution we mean a continuous function u, which satisfies (1), defined on the maximal interval of its existence $[0, T)$, where $T > 0$, such that $u(0) = 0$ and $u(t) > 0$ for $t \in (0, T)$. If additionally u satisfies $\lim_{t\to T^-} u(t) = \infty$ for some $T < \infty$, we say that u is a blow-up solution of (1).

Among the many conditions obtained for the existence of nontrivial or blow-up solutions, we can distinguish in particular two kinds of such conditions: those expressed in terms of the convergence of some series [3,4] and those expressed in terms of the convergence of some integrals [5–10]. However, there is quite a big difference between these two classes of conditions. Unlike the former, the latter are always applicable to (1) only with fixed or monotone kernel k or with fixed nonlinearity g. In this article new integral conditions for the existence of nontrivial and blow-up solutions are established without this defect. Moreover, surprisingly, they are connected with the famous Schröder functional equation.

2. Basic assumptions and facts

Throughout this paper we assume that in (1) the functions k and g satisfy the following conditions:
k is a locally integrable function such that $k(x) > 0$ for $x > 0$,

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