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Common fixed points for $\mathcal{JH}$-operators and occasionally weakly biased pairs under relaxed conditions

N. Hussain [a,*], M.A. Khamsi [b,c], A. Latif [a]

[a] Department of Mathematics, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

[b] Department of Mathematical Sciences, The University of Texas at El Paso, El Paso, TX 79968, USA

[c] Department of Mathematics and Statistics, King Fahd University of Petroleum & Minerals, P.O. Box 411, Dhahran 31261, Saudi Arabia

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ABSTRACT

Some common fixed point theorems due to Bhatt et al. [A. Bhatt, et al., Common fixed point theorems for occasionally weakly compatible mappings under relaxed conditions, Nonlinear Anal. 73 (2010) 176–182], Jungck and Rhoades [G. Jungck and B. E. Rhoades, Fixed point theorems for occasionally weakly compatible mappings, Fixed Point Theory 7 (2) (2006) 287–296. Fixed Point Theory 9 (2008) 383–384 (erratum)] and Imdad and Soliman [M. Imdad, A.H. Soliman, Some common fixed point theorems for a pair of tangential mappings in symmetric spaces, Appl. Math. Lett. 23 (2010) 351–355] are extended to two new classes of non-commuting selfmaps which contain the occasionally weakly compatible and weakly biased selfmaps as proper subclasses. Some illustrative examples are also provided to highlight the realized improvements.

1. Introduction and preliminaries

The study of common fixed points of mappings satisfying certain contractive conditions has been the focus of vigorous research activity. In 1976, Jungck [1], proved a common fixed point theorem for commuting maps, generalizing the Banach contraction principle. Sessa [2] introduced the notion of weakly commuting maps. Jungck [3] coined the term compatible mappings in order to generalize the concept of weak commutativity and showed that weakly commuting maps are compatible but the converse is not true. Pant [4] defined pointwise R-weakly commuting maps and proved common fixed point theorems, assuming the continuity of at least one of the mappings. Jungck [5] defined a pair of selfmappings to be weakly compatible if they commute at their coincidence points. In recent years, several authors have obtained coincidence point results for various classes of mappings on a metric space, utilizing these concepts. Jungck and Pathak [6] defined the concept of the weakly biased maps in order to generalize the concept of weak compatibility.

The set of fixed points of T (resp. f) is denoted by $F(T)$ (resp. $F(f)$). A point $x \in M$ is a coincidence point (common fixed point) of f and T if $fx = Tx (x = fx = Tx)$. Maps $f, T : X \to X$ are called (1) commuting if $Tfx = fTx$ for all $x \in X$, (2) R-weakly commuting [4] if for all $x \in X$, there exists $R > 0$ such that $\|fTx - Tfx\| \leq R\|fx - Tx\|$. If $R = 1$, then the maps are called weakly commuting; (3) compatible [3] if $\lim_n \|Tfx_n - fTx_n\| = 0$ when $\{x_n\}$ is a sequence such that $\lim_n Tx_n = \lim_n fx_n = t$

* Corresponding author. Tel.: +966 2 6331085.
E-mail addresses: nhusain@kau.edu.sa, hussainjam@hotmail.com (N. Hussain), mohamed@math.utep.edu, mkhamsi@kfupm.edu.sa (M.A. Khamsi), alatif@kau.edu.sa (A. Latif).

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