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Global existence and uniform decay of a damped Klein–Gordon equation in a noncylindrical domain

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ABSTRACT

In this paper, we consider a damped Klein–Gordon equation in a noncylindrical domain. This work is devoted to proving the existence of global solutions and decay for the energy of solutions for a damped Klein–Gordon equation in a noncylindrical domain.

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1. Introduction

Let Ω be an open bounded domain of $\mathbb{R}^n$ containing the origin and having C^2 boundary. Let $\gamma : [0,\infty[\to \mathbb{R}$ be a continuously differentiable function. Consider the family of subdomains $\{\Omega_t\}_{0\le t<\infty}$ of $\mathbb{R}^n$ given by $\Omega_t = T(\Omega), T : y \in \Omega \mapsto x = \gamma(t)y$, whose boundaries are denoted by Γ_t, and let $\hat{Q}$ be the noncylindrical domain of $\mathbb{R}^{n+1}$ given by

$$\hat{Q} = \bigcup_{0\le t<\infty} \Omega_t \times \{t\}$$

with boundary

$$\hat{\Sigma} = \bigcup_{0\le t<\infty} \Gamma_t \times \{t\}.$$

In this paper, we are concerned with global existence and uniform decay of the energy to a damped Klein–Gordon equation given by

$$\begin{cases} u'' + au' - b\Delta u + k|u|^\rho u = f & \text{in } \hat{Q}, \\ u = 0 \quad \text{on } \hat{\Sigma}, \\ u(x,0) = u_0, \qquad u'(x,0) = u_1 & \text{in } \Omega_0, \end{cases} \tag{1.1}$$

where $b \ge 1$, a and k are positive constants and ρ is a nonnegative constant. Δ stands for the Laplacian with respect to the spatial variables; $'$ denotes the derivative with respect to time t.

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