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H^2-boundedness of the pullback attractors for non-autonomous 2D Navier–Stokes equations in bounded domains

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ABSTRACT

We prove some regularity results for the pullback attractors of a non-autonomous 2D Navier–Stokes model in a bounded domain Ω of $\mathbb{R}^2$. We establish a general result about $(H^2(\Omega))^2 \cap V$-boundedness of invariant sets for the associate evolution process. Then, as a consequence, we deduce that, under adequate assumptions, the pullback attractors of the non-autonomous 2D Navier–Stokes equations are bounded not only in V but also in $(H^2(\Omega))^2$.

1. Introduction and setting of the problem

Let us consider the following problem for a non-autonomous 2D Navier–Stokes system:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \nu \triangle u + (u \cdot \nabla)u + \nabla p = f(t) \quad \text{in } \Omega \times (\tau, +\infty), \\ \nabla \cdot u = 0 \quad \text{in } \Omega \times (\tau, +\infty), \\ u = 0 \quad \text{on } \partial\Omega \times (\tau, +\infty), \\ u(x, \tau) = u_\tau(x), \quad x \in \Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^2$ is a bounded open set, with a regular boundary $\partial\Omega$, the number $\nu > 0$ is the kinematic viscosity, u is the velocity field of the fluid, p is the pressure, $\tau \in \mathbb{R}$ is a given initial time, u_τ is the initial velocity field, and $f(t)$ is a given external force field.

To set our problem in the abstract framework, we consider the following usual abstract spaces (see [1–4]):

$$\mathcal{V} = \{u \in (C_0^\infty(\Omega))^2 : \operatorname{div} u = 0\},$$

$H =$ the closure of $\mathcal{V}$ in $(L^2(\Omega))^2$ with inner product $(\cdot, \cdot)$ and associate norm $|\cdot|$, where for $u, v \in (L^2(\Omega))^2$,

$$(u, v) = \sum_{j=1}^{2} \int_\Omega u_j(x) v_j(x) \mathrm{d}x,$$

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