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Single and multi-peak solutions for a nonlinear Maxwell–Schrödinger system with a general nonlinearity

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ABSTRACT

For the following elliptic system in $\mathbb{R}^3$

$$-\varepsilon^2 \Delta v + V(x)v + \phi(x)v = f(v),$$
$$-\Delta\phi = v^2, \qquad \lim_{|x|\to\infty} \phi(x) = 0,$$

we construct positive solutions u and ϕ which concentrate at several given isolated local minimum components of V as $\varepsilon \to 0$. The potential V is a strictly positive continuous function and the nonlinearity f is subcritical near infinity and superlinear near zero and satisfies only the Berestycki–Lions condition.

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1. Introduction and main results

We deal with the semiclassical state of following system of the Maxwell–Schrödinger equation,

$$\begin{cases} \varepsilon^2 \Delta v - V(x)v - \phi(x)v + f(v) = 0 & \text{in } \mathbb{R}^3 \\ \Delta\phi + v^2 = 0, \quad \lim_{|x|\to\infty} \phi(x) = 0 & \text{in } \mathbb{R}^3. \end{cases} \tag{1}$$

In quantum physics, this system describes a system of many particles interacting with an electromagnetic field. For the detailed physical motivations and derivations, we refer [1–3] and references therein.

Recently, system (1) has been studied extensively under various conditions on f, especially when $f \equiv |v|^{p-1}v$. In the case of $\varepsilon = 1$, $V(x) \equiv 1$ and $f \equiv |v|^{p-1}v$, Coclite [4], D'Aprile–Mugnai [5] and Ruiz [6] obtained radially symmetric positive solutions of (1) on the range of $p \in (2, 5)$. Azzollini–Pomponio [7] considered ground state solutions on the same range of p. Ruiz [6] also proved a nonexistence result on the range $p \in (1, 2)$. For the multiplicity results, see [8].

In the case of $\varepsilon = 1$ and $V \not\equiv$ being constant, Azzollini–Pomponio [7] and Zhao–Zhao [9] showed under certain conditions on V the existence of ground state solutions of (1) with $f = |v|^{p-1}v$, $p \in (2, 5)$. When V and f are periodic Zhao–Zhao [9] also obtained mulipicity results. Non-periodic cases were dealt with by Chen and Tang [10].

Our real concern is that the semiclassical state, i.e, ε is sufficiently small. In case that V is constant and $f \equiv |v|^{p-1}v$, $p \in (1, \frac{11}{7})$, D'Aprile [11] and Ruiz [12] obtained a positive solution of (1) concentrating a whole sphere in $\mathbb{R}^3$ as $\varepsilon \to 0$ provided the second equation $\Delta\phi + v^2 = 0$ is replaced by $\Delta\phi + \frac{1}{\varepsilon}v^2 = 0$. Ruiz [12] also showed that the range $p \in (1, \frac{11}{7})$ is necessary. In case that V is not constant, Ding et al. [2] obtained existence and multiplicity results for sufficiently small $\varepsilon > 0$ under some conditions of f as the following:

$(\widetilde{\text{F1}})$ $f \in C(\mathbb{R}, \mathbb{R})$ and $f(t) = o(t)$ as $t \to 0$.

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