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Coupled fixed point theorems for multi-valued nonlinear contraction mappings in partially ordered metric spaces

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ABSTRACT

In this paper, we establish two coupled fixed point theorems for multi-valued nonlinear contraction mappings in partially ordered metric spaces. The theorems presented extend some results due to Ćirić (2009) [3]. An example is given to illustrate the usability of our results.

1. Introduction

Let (X, d) be a metric space. We denote by $CB(X)$ the collection of non-empty closed bounded subsets of X. For $A, B \in CB(X)$, and $x \in X$, suppose that

$$D(x, A) = \inf_{a \in A} d(x, a)$$

and

$$H(A, B) = \max\{\sup_{a \in A} D(a, B), \sup_{b \in B} D(b, A)\}.$$

Such a mapping H is called a Hausdorff metric in $CB(X)$ induced by d.

Definition 1.1. An element $x \in X$ is said to be a fixed point of a set-valued mapping $T : X \to CB(X)$ if and only if $x \in Tx$.

The existence of fixed points for various multi-valued contractive mappings has been studied by many authors under different conditions. For details, we refer the reader to [1–10] and the references therein. In 1969, Nadler [11] extended the famous Banach Contraction Principle [12] from single-valued mapping to multi-valued mapping and proved the following fixed point theorem for the multi-valued contraction.

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