Nearly strict convexity in Musielak–Orlicz–Bochner function spaces☆

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ABSTRACT

Criteria for nearly strict convexity of Musielak–Orlicz–Bochner function spaces equipped with the Luxemburg norm are given. We also prove that, in Musielak–Orlicz–Bochner function spaces generated by strictly convex Banach space, nearly strict convexity and strict convexity are equivalent.

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1. Introduction

In 1988, Sekowski and Stachura [1] introduced the notion of nearly strict convexity of Banach spaces by means of the Kuratowski measure of non-compactness. Baitao Song and Chaoxun Nan [2] discussed the best approximation in nearly strictly convex Banach spaces. They proved that a Banach space X is nearly strictly convex space if and only if all subspaces of X are compact-semi-Chebyshev subspaces, i.e. for any $x \in X$, $P_E(x)$ is a compact set, where $P_E(x) = \{z \in E : \|x - z\| = \inf_{y \in E} \|x - y\|\}$. This shows that the notion of nearly strictly convex space is an extension of the notion of strictly convex Banach spaces. Donghai Gi and Tingfu Wang [4] proved that nearly strict convexity and strict convexity are equivalent in Orlicz function spaces equipped with the Luxemburg norm.

Let $(X, \|\cdot\|)$ be a real Banach space. $S(X)$ and $B(X)$ denote the unit sphere and unit ball of X, respectively. By X^* denote the dual space of X. Let N, R and R^+ denote the set of natural number, reals and nonnegative reals, respectively. Let $A(x) = \{f : f \in S(X^*), f(x) = 1 = \|x\|\}$, $A_f = \{x : x \in S(X), f(x) = 1 = \|f\|\}$.

The aim of this paper is to give criteria for nearly strict convexity of Musielak–Orlicz–Bochner function spaces equipped with the Luxemburg norm. Moreover, we give a sufficient condition that nearly strict convexity implies strict convexity. The topic of this paper is related to the topic of [2–19].

2. Preliminaries

First let us recall some geometrical concepts. Consider a convex subset A of a Banach space X. A point $x \in A$ is said to be an extreme point of A if $2x = y + z$ and $y, z \in A$ imply $y = z$. The set of all extreme points of A is denoted by $ExtA$. If $ExtB(X) = S(X)$, then X is said to be strictly convex space.