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New results on the asymptotic behavior of solutions to a class of second order nonhomogeneous difference equations

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ABSTRACT

We investigate the asymptotic behavior of solutions to the following system of second order nonhomogeneous difference equation:

$$\begin{cases} u_{n+1} - (1+\theta_n)u_n + \theta_n u_{n-1} \in c_n A u_n + f_n & n \geq 1 \\ u_0 = x, \quad \sup\limits_{n\geq 0} |u_n| < +\infty \end{cases}$$

where A is a maximal monotone operator in a real Hilbert space H, $\{c_n\}$ and $\{\theta_n\}$ are positive real sequences and $\{f_n\}$ is a sequence in H. We show the weak and strong convergence of solutions and their weighted averages to an element of $A^{-1}(0)$, which is the asymptotic center of the sequence $\{u_n\}$, under appropriate assumptions on the sequences $\{c_n\}$, $\{\theta_n\}$ and $\{f_n\}$. Our results continue our previous work in Djafari Rouhani and Khatibzadeh (2008, 2010) [30,31], by presenting some new results on the asymptotic behavior of solutions, including in particular a completely new strong convergence result, and extend some previous results by Apreutesei (2003) [27,28], Morosanu (1979) [21] and Mitidieri and Morosanu (1985–86) [22] to the nonhomogeneous case and without assuming A to have a nonempty zero set.

1. Introduction

Let H be a real Hilbert space with inner product $(.,.)$ and norm $|.|$. We denote weak convergence in H by $\rightharpoonup$ and strong convergence by $\to$. A (nonlinear) possibly multivalued operator in H is a nonempty subset A of $H \times H$. A is said to be monotone (resp. strongly monotone) if $(y_2 - y_1, x_2 - x_1) \geq 0$ (resp. $(y_2 - y_1, x_2 - x_1) \geq \alpha |x_1 - x_2|^2$ for some $\alpha > 0$) for all $[x_i, y_i] \in A,\ i = 1, 2$. A is maximal monotone if A is monotone and $R(I + A) = H$, where I is the identity operator on H.

Existence, asymptotic behavior, as well as applications to optimization problems for first order nonlinear evolution equations of monotone type were studied by many authors. The discrete analogue of these equations provides an algorithm, called the proximal point algorithm, for the approximation of a solution to the monotone stationary equation $0 \in A(x)$. We refer the interested reader to [1–4] and the references therein.

Nonlinear second order evolution equations of the form

$$\begin{cases} p(t)u''(t) + r(t)u'(t) \in Au(t) + f(t) & \text{a.e. on } \mathbb{R}^+ \\ u(0) = u_0, \qquad \sup\limits_{t\geq 0} |u(t)| < +\infty \end{cases} \tag{1}$$

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