Nonlinear Analysis 74 (2011) 937–945

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

General decay to a von Kármán system with memory

C.A. Raposo [a], M.L. Santos [b,*]

[a] Departamento de Matemática, Universidade Federal de São João del-Rei, UFSJ, Frei Orlando 170, Cep 36300-000, São João del-Rei, MG, Brazil

[b] Faculdade de Matemática-PPGME, Universidade Federal do Pará, UFPA, Rua Augusto Corrêa 01, CEP 66075-110, Pará, Brazil

ARTICLE INFO

Article history:
Received 20 April 2009
Accepted 21 September 2010

Keywords:
von Kármán system
Memory dissipation
General decay

ABSTRACT

In this paper we study the von Kármán plate model with long-range memory and we show the general decay of the solution as time goes to infinity. This result generalizes and improves on earlier ones in the literature because it allows certain relaxation functions which are not necessarily of exponential or polynomial decay.

1. Introduction

The main purpose of this work is to study the general decay of the solutions to a von Kármán system for the plate equation in the presence of long-range memory. To introduce this model we need some notation. Let Ω be an open bounded set of $\mathbb{R}^2$ with regular boundary $\Gamma = \Gamma_0 \cup \Gamma_1$, where $\bar{\Gamma}_0 \cap \bar{\Gamma}_1 = \emptyset$ and $\Gamma_0 \neq \emptyset$. Let us denote by $\nu = (\nu_1, \nu_2)$ the external unit normal to Γ, and by $\eta = (-\nu_2, \nu_1)$ the unitary tangent positively oriented on Γ. By the brackets $[\cdot, \cdot]$ we denote the operator given by

$$[u, v] := \frac{\partial^2 u}{\partial^2 x}\frac{\partial^2 v}{\partial^2 y} - 2\frac{\partial^2 u}{\partial x \partial y}\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 u}{\partial^2 y}\frac{\partial^2 v}{\partial^2 x}.$$

The equations which describe small vibrations of a thin homogeneous isotropic plate of uniform thickness h are given by

$$u_{tt} - h\Delta u_{tt} + \Delta^2 u - \int_0^t g(t-\tau)\Delta^2 u(\tau)\,\mathrm{d}\tau = [u, v] \quad \text{in } \Omega \times (0, \infty), \tag{1}$$

$$\Delta^2 v = -[u, u] \quad \text{in } \Omega \times (0, \infty), \tag{2}$$

$$u(x, y, 0) = u_0(x, y), \qquad u_t(x, y, 0) = u_1(x, y) \quad \text{in } \Omega, \tag{3}$$

with the boundary conditions

$$v = \frac{\partial v}{\partial \nu} = 0 \quad \text{on } \Gamma \times (0, \infty), \tag{4}$$

$$u = \frac{\partial u}{\partial \nu} = 0 \quad \text{on } \Gamma_0 \times (0, \infty), \tag{5}$$

$$\mathcal{B}_1 u - \mathcal{B}_1 \left\{ \int_0^t g(t-\tau) u(\tau)\,\mathrm{d}\tau \right\} = 0 \quad \text{on } \Gamma_1 \times (0, \infty), \tag{6}$$

* Corresponding author.
E-mail addresses: raposo@ufsj.edu.br (C.A. Raposo), ls@ufpa.br (M.L. Santos).

0362-546X/$ – see front matter
doi:10.1016/j.na.2010.09.047