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Characterizations of reproducing cones and uniqueness of fixed points☆

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ABSTRACT

The purpose of this paper is to discuss the existence and uniqueness of fixed point in a partially ordered Banach space. Based on the characterizations of reproducing cones, some fixed point theorems for nonlinear operators are proved. As an application, the existence and uniqueness of periodical solution for a first order differential equation is discussed.

1. Introduction

Recently in [1], an analogue of Banach's fixed point theorem in a partially ordered metric space has been proved.

Theorem 1.1. *Let X be a partially ordered set such that every pair $x, y \in X$ has a lower bound and an upper bound. Furthermore, let d be a metric on X such that (X, d) is a complete metric space. If T is a continuous, monotone (i.e., either order-preserving or order-reversing) map from X into X such that*

(1) $\exists\, 0 < c < 1 : d(T(x), T(y)) \le cd(x, y),\ \forall x \ge y;$
(2) $\exists\, x_0 \in X : x_0 \le T(x_0)$ *or* $x_0 \ge T(x_0)$.

Then, T has a unique fixed point $\bar{x}$. Moreover, for every $x \in X$,

$$\lim_{n\to\infty} T^n(x) = \bar{x}.$$

Theorem 1.1 was successfully applied to establish some solvability results for matrix equations.

Since then several authors considered the problem of existence (and uniqueness) of a fixed point for contraction type operators on partially ordered sets. In 2005, Nieto and Rodriguez-Lopez proved a modified variant of Theorem 1.1, by removing the continuity of T. Their result (see [2, Theorem 2.3]) is the following.

Theorem 1.2. *Let X be a partially ordered set such that every pair $x, y \in X$ has a lower or an upper bound. Let d be a metric on X such that the metric space (X, d) is complete. Let $T : X \to X$ be an increasing operator. Suppose that the following three*

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