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Remarks on the existence of three solutions for the $p(x)$-Laplacian equations

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ABSTRACT

In this paper we improve some results on the existence of three solutions for the $p(x)$-Laplacian equations via an abstract result recently obtained by Ricceri in [2].

1. Introduction

In this paper we consider the existence of three solutions of the following $p(x)$-Laplacian equations:

$$\begin{cases} -\text{div}(|\nabla u|^{p(x)-2}\nabla u) + |u|^{p(x)-2}u = \lambda f(x,u) + \mu g(x,u) & \text{in } \Omega, \\ Bu = 0 \quad \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset R^N$ $(N \geq 2)$ is a bounded domain with smooth boundary $\partial\Omega$, $p \in C(\overline{\Omega})$, $1 < p^- := \inf_{x\in\overline{\Omega}} p(x) \leq p^+ := \sup_{x\in\overline{\Omega}} p(x) < +\infty$, $\lambda > 0$ and μ are constants.

$Bu = 0$ denotes the following boundary conditions:

(1) $B = B_1$, Dirichlet boundary condition, i.e.

$$u = 0 \quad \text{on } \partial\Omega.$$

(2) $B = B_2$, Neumann boundary condition, i.e.

$$\frac{\partial u}{\partial \gamma} = 0 \quad \text{on } \partial\Omega,$$

where γ is the outward unit normal to $\partial\Omega$.

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