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The existence of solutions to boundary value problems of fractional differential equations at resonance[☆]

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ABSTRACT

By using the coincidence degree theory due to Mawhin and constructing suitable operators, we study the existence of solutions to boundary value problems of fractional differential equations at resonance with *dimker* $L = 2$. An example is given to illustrate our result.

1. Introduction

Boundary value problems for integer order differential equations at resonance have been studied in many papers. We refer the readers to [1–15] and references cited therein. Motivated by the excellent results of [1,16,17], in this paper, we investigate the existence of solutions to the fractional differential equation at resonance:

$$D_{0+}^{\alpha}u(t) = f(t, u(t), D_{0+}^{\alpha-1}u(t)), \quad \text{a.e. } t \in [0, 1], \tag{1.1}$$

$$u(0) = 0, \quad D_{0+}^{\alpha-1}u(0) = \sum_{i=1}^{m} a_i D_{0+}^{\alpha-1}u(\xi_i), \qquad D_{0+}^{\alpha-2}u(1) = \sum_{j=1}^{n} b_j D_{0+}^{\alpha-2}u(\eta_j), \tag{1.2}$$

where $2 < \alpha < 3$, $0 < \xi_1 < \xi_2 < \cdots < \xi_m < 1$, $0 < \eta_1 < \eta_2 < \cdots < \eta_n < 1$, $\sum_{i=1}^{m} a_i = 1$, $\sum_{j=1}^{n} b_j = 1$, $\sum_{j=1}^{n} b_j\eta_j = 1$, $f : [0, 1] \times R \times R \to R$ satisfies the Carathéodory condition.

Fractional differential equations arise in a variety of different areas such as rheology, fluid flows, electrical networks, viscoelasticity, chemical physics, etc. (see [18,19] and references cited therein). Recently, boundary value problems for fractional differential equations at nonresonance have been studied by many authors (see [20,21,16,17,22–29]). More recently, Kosmatov studied the boundary value problems for fractional differential equations at resonance with *dimker* $L = 1$ (see [30]). As far as we know, boundary value problems for fractional differential equations at resonance with *dimker* $L = 2$ have not been studied. We will fill this gap in the literature.

In this paper, we will always suppose that the following conditions hold:

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