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The monotonic property and stability of solutions of fractional differential equations☆

Sung Kyu Choi, Namjip Koo *

Department of Mathematics, Chungnam National University, Daejeon 305-764, Republic of Korea

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ABSTRACT

In this paper we improve on the monotone property of Lemma 1.7.3 in Lakshmikantham et al. (2009) [5] for the case $g(t, u) = \lambda u$ with a nonnegative real number λ. We also investigate the Mittag-Leffler stability of solutions of fractional differential equations by using the fractional comparison principle.

1. Introduction

Fractional calculus is a generalization of the ordinary differentiation and integration to arbitrary non-integer order. Recently, the fractional-order differential equations have played a significant role in modelling the anomalous dynamics of various processes related to complex systems in most areas of science and engineering. Lakshmikantham et al. [1–5] have investigated the basic theory of initial value problems for fractional differential equations involving Riemann–Liouville differential operators of order $0 < q < 1$. They followed the classical approach of the theory of differential equations of integer order, in order to compare and contrast the differences as well as the intricacies that might result in development [6, Vol. I].

There exist some properties of fractional calculus that do not parallel ones of differential calculus. Let D be one of the Dini derivatives and suppose that $m \in C([a, b], \mathbb{R})$. Lakshmikantham et al. showed that if $Dm(t) \geq 0$, then $m(a) \leq m(b)$, i.e., $m(t)$ is nondecreasing in t [5, Lemma 1.7.1]. Furthermore, they extended this result to fractional differential equations involving the Caputo derivative with order $0 < q < 1$. The comparison principle is fundamental in the investigation of the properties of solutions for fractional differential equations. Thus, they applied the monotone property of solutions of nonnegative scalar Caputo fractional differential equations to derive the fractional comparison principle and the global existence of solutions for the fractional differential equations in [2,5]. Also, we have found that the monotone property

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* Corresponding author. Fax: +82 42 823 1856.
E-mail addresses: sgchoi@cnu.ac.kr (S.K. Choi), njkoo@cnu.ac.kr (N. Koo).

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