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Fixed point theorems for some generalized multivalued nonexpansive mappings

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ABSTRACT

In this paper, we introduce a condition on multivalued mappings which is a multivalued version of condition (C_λ) defined by Garcia-Falset et al. (2011) [3]. It is shown here that some of the classical fixed point theorems for multivalued nonexpansive mappings can be extended to mappings satisfying this condition. Our results generalize the results in Lim (1974), Lami Dozo (1973), Kirk and Massa (1990), Garcia-Falset et al. (2011), Dhompongsa et al. (2009) and Abkar and Eslamian (2010) [4–6,3,7,8] and many others.

1. Introduction

A mapping T on a subset E of a Banach space X is said to be a *contraction* if there exists a constant $k \in [0, 1)$ such that

$$\|Tx - Ty\| \le k\|x - y\|, \quad \text{for all } x, y \in E. \tag{1}$$

If (1) is valid when $k = 1$, then T is called *nonexpansive*. It is called *quasinonexpansive* if $\|Tx - y\| \le \|x - y\|$ for all $x \in E$ and for all $y \in F(T)$, where $F(T)$ is the set of fixed points of T.

In order to characterize the completeness of underlying metric spaces, Suzuki [1] introduced a weaker notion of contractions and proved the following theorem.

Theorem 1.1. *Define a nonincreasing function θ from $[0, 1)$ onto $(\frac{1}{2}, 1]$ by*

$$\theta(r) = \begin{cases} 1 & \text{if} \quad 0 \le r \le (\sqrt{5} - 1)/2, \\ (1 - r)r^{-2} & \text{if} \quad (\sqrt{5} - 1)/2 \le r \le 2^{-1/2}, \\ (1 + r)^{-1} & \text{if} \quad 2^{-1/2} \le r < 1. \end{cases}$$

Then for a metric space (M, d), the following are equivalent:

(i) *M is complete.*
(ii) *There exists $r \in (0, 1)$ such that every mapping T on X satisfying the following has a fixed point:*
   - *$\theta(r)d(x, Tx) \le d(x, y)$ implies $d(Tx, Ty) \le rd(x, y)$ for all $x, y \in X$.*

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