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The Tikhonov regularization extended to equilibrium problems involving pseudomonotone bifunctions ☆

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ABSTRACT

We extend the Tikhonov regularization method widely used in optimization and monotone variational inequality studies to equilibrium problems. It is shown that the convergence results obtained from the monotone variational inequality remain valid for the monotone equilibrium problem. For pseudomonotone equilibrium problems, the Tikhonov regularized subproblems have a unique solution only in the limit, but any Tikhonov trajectory tends to the solution of the original problem, which is the unique solution of the strongly monotone equilibrium problem defined on the basis of the regularization bifunction.

1. Introduction

Throughout this article we assume that K is a nonempty closed convex subset of the real Euclidean space $\mathbb{R}^n$ and $f : K \times K \to \mathbb{R}$ is an equilibrium bifunction, i.e. $f(x, x) = 0$ for every $x \in K$. We consider the following equilibrium problem which is often called the Ky Fan inequality due to the contribution of Ky Fan to this field:

$$\text{Find } x \in K \text{ such that } f(x, y) \geq 0 \quad \forall y \in K. \tag{1}$$

As usual, the problem that is the *dual* of (1) is defined as follows:

$$\text{Find } x \in K \text{ such that } f(y, x) \leq 0 \quad \forall y \in K. \tag{2}$$

We will refer to (1) and (2) as $\mathrm{EP}(K, f)$ and $\mathrm{DEP}(K, f)$, and their solution sets are denoted by $\mathrm{SEP}(K, f)$ and $\mathrm{SDEP}(K, f)$ respectively.

The equilibrium problem (1) contains as special cases some classes of optimization, Nash equilibria, Kakutani fixed points and variational inequality (see e.g. [1–3]). It unifies these problems in a convenient way, and many of the results obtained for one of these problems can be extended, with suitable modifications, to equilibrium problems. Recently, some practical models of interest in economics and engineering have been formulated as an equilibrium problem of the form (1) (see e.g. [2,4,5]). This explains the vast and increasing attention devoted to this subject.

The Tikhonov regularization is a well-known method that is widely used in convex optimization and monotone variational inequality studies to handle ill-posed problems. The main idea of the Tikhonov regularization method for the monotone variational inequality is that it adds a strongly monotone operator depending on a regularization parameter to the monotone cost operator to obtain a strongly monotone variational inequality. The resulting regularized problem then

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