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Existence and multiplicity of homoclinic solutions for a class of damped vibration problems[☆]

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ABSTRACT

The main purpose of this paper is to study the following damped vibration problems

$$\begin{cases} -\ddot{u}(t) - B\dot{u}(t) + A(t)u(t) = \nabla F(t, u(t)) \quad \text{a.e. } t \in R \\ u(t) \to 0, \qquad \dot{u}(t) \to 0 \quad \text{as } |t| \to \infty \end{cases} \tag{1.1}$$

where $A = [a_{i,j}(t)] \in C(R, R^{N^2})$ is an $N \times N$ symmetric matrix-valued function, $B = [b_{ij}]$ is an antisymmetry $N \times N$ constant matrix, $F \in C^1(R \times R^N, R)$ and $\nabla F(t, u) := \nabla_u F(t, u)$. By a symmetric mountain pass theorem and a generalized mountain pass theorem, an existence result and a multiplicity result of homoclinic solutions of (1.1) are obtained.

1. Introduction and preliminaries

Consider the following damped vibration problems

$$\begin{cases} -\ddot{u}(t) - B\dot{u}(t) + A(t)u(t) = \nabla F(t, u(t)) \quad \text{a.e. } t \in R \\ u(t) \to 0, \qquad \dot{u}(t) \to 0 \quad \text{as } |t| \to \infty \end{cases} \tag{1.1}$$

where $A = [a_{i,j}(t)] \in C(R, R^{N^2})$ is an $N \times N$ symmetric matrix-valued function, $B = [b_{ij}]$ is an antisymmetry $N \times N$ constant matrix and $F \in C^1(R \times R^N, R)$.

When B is a zero matrix, there are many results for problems (1.1), (for example, see [1–15]). It should be pointed out that these results were obtained under $F(t, u)$ periodic in t or under the global Ambrosetti–Rabinowitz type condition

$$0 < \mu F(t, u) \le (\nabla F(t, u), u), \quad \forall t \in R \text{ and } u \in R^N \setminus \{0\},$$

where $\mu > 2$ is a constant.

When $B \neq 0$, some of the existence and multiplicity results were obtained for periodic solutions in [16–18]. When $A(t)$ is a positive definite matrix and $F(t, u) = a(t)|u|^\gamma$ $(1 < \gamma < 2)$, the existence of nontrivial solution for problem (1.1) was studied in [19].

Moreover, in recent papers [20–23], by using critical point theory Xiao and Nieto [20,21] studied the existence of weak solutions of the following nonlinear impulsive differential equations

$$\begin{cases} -u''(t) + g(t)u'(t) + \lambda u(t) = f(t, u(t)) \quad \text{a.e. } t \in [0, T] \\ -\Delta u'(t_j) = I_j(u(t_j)), \quad j = 1, 2, \dots, p, \\ u(0) = u(T) = 0, \end{cases} \tag{1.2}$$

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