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Existence of multi-valued solutions with asymptotic behavior of Hessian equations

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ABSTRACT

In this paper, we use the Perron method to prove the existence of multi-valued solutions with asymptotic behavior at infinity of Hessian equations.

1. Introduction

In this paper, we study the multi-valued solutions of the Hessian equation

$$\sigma_k(\lambda(D^2u)) = 1, \tag{1.1}$$

where $\sigma_k(\lambda)$ denotes the kth elementary symmetric function of $\lambda = (\lambda_1, \ldots, \lambda_n)$, which is defined by

$$\sigma_k(\lambda) = \sum_{i_1<\cdots<i_k} \lambda_{i_1} \cdots \lambda_{i_k}, \quad k = 1, \ldots, n,$$

$\lambda = \lambda(D^2u)$ are the eigenvalues of the Hessian matrix D^2u. For $k = 1$ (1.1) is the Poisson equation $\Delta u = 1$, and for $k = n$ (1.1) is the Monge–Ampère equation $\det(D^2u) = 1$.

From the theory of analytic functions, we know that the typical two dimensional examples of multi-valued harmonic functions are

$$u_1(z) = \mathrm{Re}\left(z^{\frac{1}{k}}\right), \quad z \in \mathbb{C} \setminus \{0\},$$
$$u_2(z) = \mathrm{Arg}(z), \quad z \in \mathbb{C} \setminus \{0\},$$

and

$$u_3(z) = \mathrm{Re}(\sqrt{(z-1)(z+1)}), \quad z \in \mathbb{C} \setminus \{\pm 1\}.$$

By 1970s, Almgren [1] had realized that a minimal variety near a multiplicity-k disc could be well approximated by the graph of a multi-valued function minimizing a suitable analog of the ordinary Dirichlet integral. Many facts about harmonic functions are also true for these Dirichlet minimizing multi-valued functions. Evans [2–4], Levi [5] and Caffarelli [6,7] studied the multi-valued harmonic functions. Evans [3] proved that the conductor potential of a surface with minimal capacity was a double-valued harmonic function. In [7], Caffarelli proved the Hölder continuity of the multi-valued harmonic functions.

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