Nonlinear Analysis 74 (2011) 3870–3875

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

On best proximity pair theorems for relatively u-continuous mappings

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ARTICLE INFO

Article history:
Received 11 April 2010
Accepted 14 February 2011

Keywords:
Best proximity points
Relatively u-continuous mappings
Metric projection
Affine mappings
Common best proximity points

ABSTRACT

A new class of mappings, called relatively u-continuous, is introduced and used to investigate the existence of best proximity points. As an application of the existence theorem, we obtain a generalized version of the Markov–Kakutani theorem for best proximity points in the setting of a strictly convex Banach space.

1. Introduction

Let (X, d) be a metric space and consider a mapping $T : A \to X$, where A is a nonempty subset of X. The mapping T is said to have a fixed point in A if the fixed point equation $Tx = x$ has at least one solution. In metric terminology, we say that $x \in A$ is a fixed point of T if $d(x, Tx) = 0$. It is clear that the necessary condition for the existence of a fixed point for T is $T(A) \cap A \neq \emptyset$ (but not sufficient). If the fixed point equation $Tx = x$ does not possess a solution, then $d(x, Tx) > 0$ for all $x \in A$. In such a situation, it is our aim to find an element $x \in A$ such that $d(x, Tx)$ is minimum in some sense. The best approximation theory and best proximity pair theorems are studied in this direction. Consider the following well-known best approximation theorem due to Ky Fan [1].

Theorem 1.1 ([1]). *Let A be a nonempty compact convex subset of a normed linear space X and $T : A \to X$ be a continuous function. Then there exists $x \in A$ such that $\|x - Tx\| = \operatorname{dist}(Tx, A) := \inf\{\|Tx - a\| : a \in A\}$.*

Such an element $x \in A$ in Theorem 1.1 is called a best approximant of T in A. Note that if $x \in A$ is a best approximant, then $\|x - Tx\|$ need not be the optimum. Best proximity point theorems have been explored to find sufficient conditions so that the minimization problem

$$\min_{x \in A} \|x - Tx\| \tag{1}$$

has at least one solution. To have a concrete lower bound, let us consider two nonempty subsets A, B of a metric space X and a mapping $T : A \to B$. The natural question is whether one can find an element $x_0 \in A$ such that $d(x_0, Tx_0) = \min\{d(x, Tx) : x \in A\}$. Since $d(x, Tx) \geq \operatorname{dist}(A, B)$, the optimal solution to the problem of minimizing the real valued function $x \mapsto d(x, Tx)$ over the domain A of the mapping T will be the one for which the valued $\operatorname{dist}(A, B)$ is attained. A point $x_0 \in A$ is called a best proximity point of T if $d(x_0, Tx_0) = \operatorname{dist}(A, B)$. Note that if $\operatorname{dist}(A, B) = 0$, then the best proximity point is nothing but a fixed point of T.

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doi:10.1016/j.na.2011.02.021