Nonlinear Analysis 74 (2011) 4889–4897

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Tripled fixed point theorems for contractive type mappings in partially ordered metric spaces

Vasile Berinde *, Marin Borcut

Department of Mathematics and Computer Science, North University of Baia Mare, Victoriei 76, 430122 Baia Mare, Romania

ARTICLE INFO

Article history:
Received 4 February 2011
Accepted 18 March 2011
Communicated by Ravi Agarwal

Keywords:
Metric space
Contractive type operator
Tripled fixed point
Existence and uniqueness

ABSTRACT

In this paper, we introduce the concept of tripled fixed point for nonlinear mappings in partially ordered complete metric spaces and obtain existence, and existence and uniqueness theorems for contractive type mappings. Our results generalize and extend recent coupled fixed point theorems established by Gnana Bhaskar and Lakshmikantham [T. Gnana Bhaskar, V. Lakshmikantham, Fixed point theorems in partially ordered metric spaces and applications, Nonlinear Anal. 65 (7) (2006) 1379–1393]. Examples to support our new results are given.

1. Introduction

In a recent paper, Gnana Bhaskar and Lakshmikantham [1] introduced the concepts of coupled fixed point and mixed monotone property for contractive operators of the form $F : X \times X \to X$, where X is a partially ordered metric space, and then established some interesting coupled fixed point theorems. They also illustrated these important results by proving the existence and uniqueness of the solution for a periodic boundary value problem.

We summarize in the following the basic notions and results established in [1], in view of their generalization.

Definition 1 (*[1]*)**.** Let $(X, \leq)$ be a partially ordered set and d be a metric on X such that (X, d) is a complete metric space. Further, endow the product space $X \times X$ with the following partial order:for

$$(x, y), (u, v) \in X \times X, \quad (u, v) \leq (x, y) \Leftrightarrow x \geq u, y \leq v.$$

Definition 2 (*[1]*)**.** Let$(X, \leq)$ be a partially ordered set and $F : X \times X \to X$. We say that F has the mixed monotone property if $F(x, y)$ is monotone nondecreasing in x and is monotone non increasing in y, that is, for any $x, y \in X$,

$$x_1, x_2 \in X, \quad x_1 \leq x_2 \Rightarrow F(x_1, y) \leq F(x_2, y)$$

and,

$$y_1, y_2 \in X, \quad y_1 \leq y_2 \Rightarrow F(x, y_1) \geq F(x, y_2).$$

Definition 3 (*[1]*)**.** Call an element $(x, y) \in X \times X$ a coupled fixed point of the mapping F if

$$F(x, y) = x, \qquad F(y, x) = y.$$

* Corresponding author. Tel.: +40 262 427466; fax: +40 262 275368.
E-mail addresses: vberinde@ubm.ro, vasile_berinde@yahoo.com (V. Berinde), marinborcut@yahoo.com (M. Borcut).

0362-546X/$ – see front matter
doi:10.1016/j.na.2011.03.032