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Asymptotic behavior of positive solutions of a semilinear Dirichlet problem

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ABSTRACT

In this paper, we study the asymptotic behavior of the unique positive classical solution to the following semilinear boundary value problem

$$\Delta u + a(x)u^{\alpha} = 0, \quad x \in \Omega, \quad u > 0 \text{ in } \Omega, \quad u_{|\partial\Omega} = 0.$$

Here Ω is a bounded $C^{1,1}$ domain, $\alpha < 1$ and the function a is in $C^{\gamma}_{loc}(\Omega)$, $0 < \gamma < 1$ such that there exists $c > 0$ satisfying for each $x \in \Omega$,

$$\frac{1}{c} \leq a(x)\delta(x)^{\lambda} \exp\left(-\int_{\delta(x)}^{\eta} \frac{z(t)}{t} dt\right) \leq c,$$

where $\lambda \leq 2$, $\eta > d = diam(\Omega)$, $\delta(x) = dist(x, \partial\Omega)$ and z is a continuous function on $[0, \eta]$ with $z(0) = 0$.

1. Introduction

Let Ω be a bounded $C^{1,1}$- domain in $\mathbb{R}^n$, $n \geq 2$ and let $\alpha < 1$. This paper deals with estimates of positive solutions of the following semilinear boundary value problem

$$\begin{cases} \Delta u + a(x)u^{\alpha} = 0, & x \in \Omega, \\ u > 0, \quad \text{in } \Omega, & u_{|\partial\Omega} = 0. \end{cases} \tag{1.1}$$

The existence of such solutions and their behavior have been extensively investigated by many authors (see [1–10], and the references therein). For $a \equiv 1$ on Ω and $\alpha < -1$, Crandall et al. proved in [5] that problem (1.1) has a unique classical solution u in Ω which satisfies

$$c_1(\delta(x))^{\frac{2}{1-\alpha}} \leq u(x) \leq c_2(\delta(x))^{\frac{2}{1-\alpha}}, \quad \text{near the boundary } \partial\Omega, \tag{1.2}$$

where c_1, c_2 are positive constants and $\delta(x) = \text{dist}(x, \partial\Omega)$.

In [8], Lazer and Mckenna showed that (1.2) continuous to hold on $\overline{\Omega}$ and if instead of $a \equiv 1$ on Ω, they assume that $0 < b_1 \leq a(x)(\delta(x))^{\lambda} \leq b_2$, for all $x \in \overline{\Omega}$, where b_1, b_2 are positive constants and $\lambda \in (0, 2)$, then they proved that for $\alpha < -1$, there exist positive constants c_1 and c_2 such that

$$c_1(\delta(x))^{\frac{2}{1-\alpha}} \leq u(x) \leq c_2(\delta(x))^{\frac{2-\lambda}{1-\alpha}}, \quad \text{for } x \in \overline{\Omega}.$$

In [2,3], Brezis et al. studied the sublinear case (i.e. $0 < \alpha < 1$) with the condition that the function a is positive and bounded. They showed that problem (1.1) has a unique positive solution.

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