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Existence of a positive almost periodic solution for a nonlinear delay integral equation[☆]

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ABSTRACT

In this paper, we discuss the existence of positive almost periodic solution for some nonlinear delay integral equation, by constructing a new fixed point theorem in the cone. Some known results are extended.

1. Introduction

This paper is the continuity of the work [1]. Motivated by the work in [2–8,1], the author and Xu and Yuan [1] studied the following nonlinear integral equation

$$x(t) = \int_{t-\tau(t)}^{t} [f(s, x(s)) + g(s, x(s))]\mathrm{d}s, \tag{1}$$

where f, g, τ are positive almost periodic. They presented the existence and uniqueness of positive almost periodic solutions to Eq. (1) by constructing a fixed point theorem [1, Theorem 1], associated with mixed monotone operators in the cone. Set $F(t, x, y) = f(t, x) + g(t, y)$. According to [1, Theorem 1], the following important assumption imposed on the function f, g can be formulated as follows.

(H_1') There exists a positive function $\varphi : (0, 1) \to (0, 1)$ such that for every $\lambda \in (0, 1)$, one has $\varphi(\lambda) > \lambda$, and

$$F(t, \lambda x, \lambda^{-1} y) \geq \varphi(\lambda) F(t, x, y), \quad \forall t \in \mathbb{R},\ x, y > 0.$$

It is easy to note that [1, Theorem 1] can only deal with homogeneous nonlinearities, and does not work in the more general circumstance. In this paper, we would like to present a simple method, i.e., construct a new fixed point theorem in regard to the mixed monotone operators to overcome this restriction, through which, we can relax the above (H_1') to the following assumption (H_1) to present the existence and uniqueness of positive almost periodic solutions to Eq. (1).

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