



# Almost automorphic solutions for differential equations with piecewise constant argument in a Banach space

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## ARTICLE INFO

### Article history:

Received 29 October 2010

Accepted 25 November 2010

### MSC:

34K05

34A12

34A40

### Keywords:

Almost automorphic solution

Spectral theory

Differential equations with piecewise constant argument

## ABSTRACT

Using spectral theory we obtain sufficient conditions for the almost automorphy of bounded solutions to differential equations with piecewise constant argument of the form  $x'(t) = A(t)x([t]) + f(t)$ ,  $t \in \mathbb{R}$ , where  $A(t)$  is an almost automorphy operator,  $f(t)$  is an  $\mathbb{X}$ -valued almost automorphic function and  $\mathbb{X}$  is a finite dimensional Banach space.

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## 1. Introduction

The study of differential equations with piecewise constant argument (EPCA) is an important subject because these equations have the structure of continuous dynamical systems in intervals of unit length. Therefore they combine the properties of both differential and difference equations. There have been many papers studying EPCA. The first work devoted to this subject is the paper of Shah and Wiener [1] in 1983. In 1984 Cook and Wiener in [2] study these equations with delay. However, it is only in 2006 that the almost automorphy of solutions of these types of equations is considered. In their paper [3], Nguyen Van minh and Tran Tat Dat give sufficient spectral conditions for the almost automorphy of bounded solutions of the equations of the form

$$x'(t) = Ax([t]) + f(t)$$

where  $A$  is a bounded linear operator in a finite dimensional Banach space  $\mathbb{X}$ ,  $f$  is an  $\mathbb{X}$ -valued almost automorphic function on  $\mathbb{R}$  and  $[.]$  is the largest integer function. It is natural to raise the question as whether bounded solutions of the non-autonomous case

$$x'(t) = A(t)x([t]) + f(t) \tag{1}$$

are almost automorphic, where  $f(t)$  is an  $\mathbb{X}$ -valued almost automorphic function on  $\mathbb{R}$ ,  $A(t)$  is an operator in  $\mathbb{X}$ -valued almost automorphic and  $\mathbb{X}$  does not contain any subspace isomorphic to  $c_0$ ? In this paper we will prove that it is true. Consequently, we generalize the work of Nguyen Van minh and Tran Tat Dat giving sufficient spectral conditions for the almost automorphy of bounded solutions of (1).

The rest of this paper is organized as follows. First of all, we will recall the concept of almost automorphic functions and almost automorphic sequences. We recall also the concept of uniform spectrum of a bounded sequence with respect to a

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