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Banach operator pair and common fixed points for nonexpansive maps[☆]

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ABSTRACT

Let X be a normed space and T and I be two nonexpansive self-maps on the closed convex subset $C \subset X$. It is proved that if (I, T) is a Banach operator pair and $\overline{T(C)}$ is compact then $F(I, T) \neq \emptyset$. A family of nonexpansive maps is also investigated; the well-known De Marr's fixed point theorem is extended to the noncommuting case by introducing a notion of Banach operator family. As an application of the theorem, the problem for the common fixed point in invariant approximation for convex set is solved directly in a quite different way from the others before.

1. Introduction and preliminaries

Let (X, d) be a metric space and I and T be self-maps of X. A point $x \in X$ is called a fixed point of I if $Ix = x$ and a common fixed point of I and T if $Ix = Tx = x$. Denote the set of fixed points of I by $F(I)$ and the set of common fixed points of I and T by $F(I, T)$.

The problems about common fixed point for two (or a family of) maps as an important part of the fixed point theory have been studied by many authors (see [1–16]). At the early time, the common fixed point was treated as a generalization of the fixed point, so the commutativity for two maps was always assumed. But soon after it was found that the two maps were not necessarily commutative at each point, then some noncommuting classes of maps satisfying the so-called weakly commuting conditions were introduced. The followings are some well-known notions.

Let (X, d) be a metric space, $I : X \to X$, $T : X \to X$. The pair $\{I, T\}$ is called weakly commutative [13] if $d(ITx, TIx) \leq d(Ix, Tx)$ for all $x \in X$; R-weakly commuting [12] if there exists an $R > 0$ such that $d(ITx, TIx) \leq Rd(Ix, Tx)$ for all $x \in X$; compatible [8] if $\lim_{n\to\infty} d(ITx_n, TIx_n) = 0$ provided that $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ix_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$; weakly compatible if they commute at their coincidence points, i.e., if $ITx = TIx$ whenever $Ix = Tx$; occasionally weakly compatible [2] if they commute for some coincidence point of I and T.

It is noted that the class of occasionally weakly compatible maps contains the previous four ones. And the occasionally weakly compatible condition was succeeded in dealing with the common fixed point problems for two maps I and T when

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