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On cyclic Meir–Keeler contractions in metric spaces

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ABSTRACT

Cyclic Meir–Keeler contractions are considered under the recently introduced WUC and HW properties on pairs of subsets of metric spaces. We show that, in contrast with previous results in the theory, best proximity point theorems under these properties do not directly extend from cyclic contractions to cyclic Meir–Keeler contractions. We obtain, however, a positive result for cyclic Meir–Keeler contractions under additional properties which is shown to be an extension of already existing results for cyclic contractions. Moreover, we give examples supporting the necessity of our additional conditions.

1. Introduction

Let A and B be nonempty subsets of a metric space X and suppose that a mapping $T: A\cup B \to A\cup B$ is such that $T(A)\subset B$, $T(B)\subset A$ and there exists $k\in(0,1)$ for which

$$d(T(x),T(y)) \leq (1-k)d(x,y)+kd(A,B), \quad x\in A,\ y\in B.$$

Then T is called *a cyclic contraction*. In [1], it was first stated that cyclic contractions in uniformly convex Banach spaces have unique best proximity points, i.e. points such that $d(x,T(x)) = d(A,B)$, under the assumption that A and B are convex and closed. Moreover, Eldred and Veeramani proved in [1] that every sequence $(T^{2n}(x))$ of Picard iterates tends to this point. Next the problem of the existence of best proximity points and the convergence of iterates was further studied by different authors (see e.g. [2–8]).

Especially, in [2] similar results were obtained in complete metric spaces. There the properties were transferred from the space to the pair (A,B) of considered sets. More precisely, *UC property* (see next section for definition) for a pair of subsets (A,B) of a metric space was introduced. In that same paper the authors proved the following generalization of the best proximity point theorem for cyclic Meir–Keeler contractions (compare [4, Theorem 3.10] and [9]).

Theorem 1.1 *([2, Theorem 3]).* *Let (X,d) be a metric space and let A and B be nonempty subsets of X such that (A,B) satisfies the UC property and A is complete. Suppose that $T: A\cup B\to A\cup B$ satisfy $T(A)\subset B$, $T(B)\subset A$ and for each $\varepsilon>0$ there is $\delta>0$ such that*

$$d(x,y) < d(A,B)+\varepsilon+\delta \Rightarrow d(T(x),T(y)) < d(A,B)+\varepsilon, \quad x\in A,\ y\in B.$$

Then

(a) *T has a unique best proximity point $z\in A$;*
(b) *z is a fixed point of T^2;*
(c) *for each $x\in A$ the sequence $(T^{2n}(x))$ tends to z.*

Also very recently Espínola and Fernández-León [7] have shown that the best proximity point theorem for cyclic contractions remains true if the assumption of property UC is weakened by the new property WUC (see [7, Theorem 3.20])

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